

Exploring Relative Size with Relative Risk

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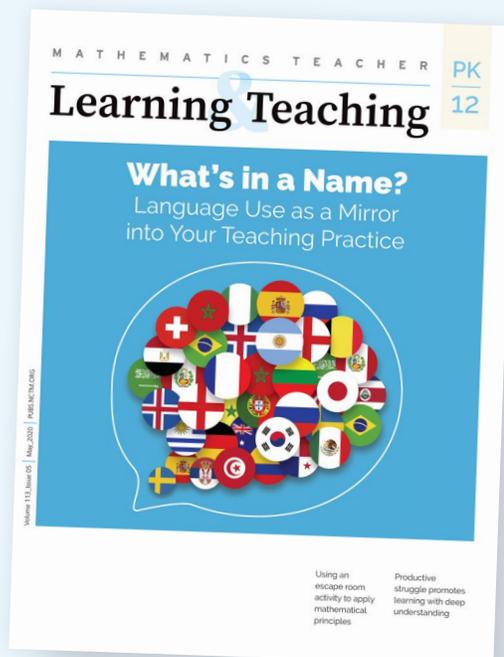
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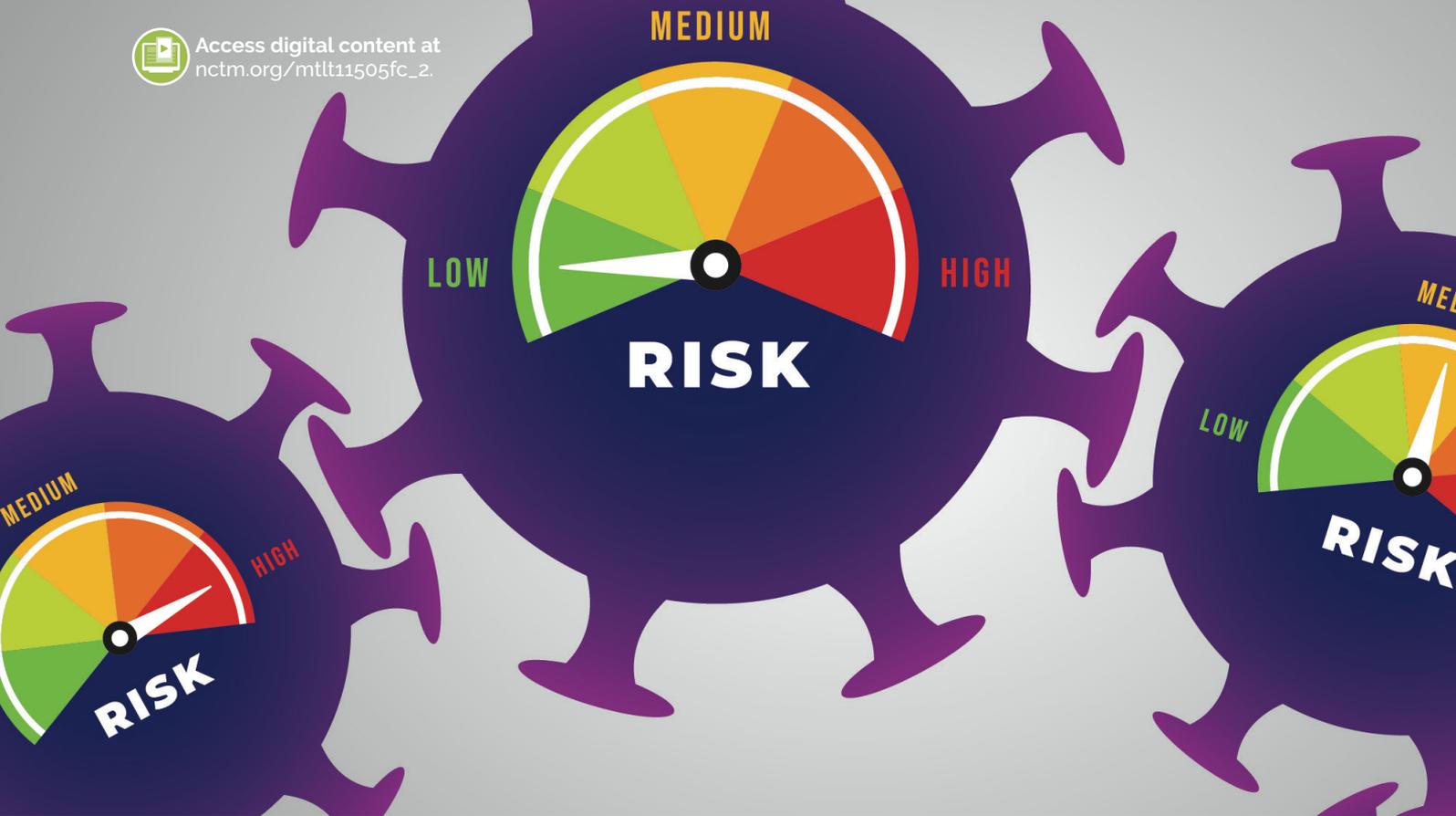
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Exploring Relative Size with Relative Risk

The Relative Risk Tool web app allows students to compare risks relating to COVID-19 with other more familiar risks, to make multiplicative comparisons, and to interpret them.

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Alexandra Yon, Cameron Byerley, Hyunkyong Yoon, and Kevin Moore

Disease has been within the fabric of society for centuries, and an increasing knowledge of disease has brought about an awareness of the range of impact that disease imparts. From plant life to animal life, disease has the capability to ravage crops, expose inequities, and reshape educational systems. During the 2020–2021 academic year, the COVID-19 pandemic affected the lives of K–12 students in radical ways as their parents, teachers, and communities made decisions that contributed to COVID-19's impact on the students' lives. Many of these students experienced poverty, virtual schooling, and other stressors that were out of their control (Tang, Cheung, and

Xiang 2021), although some students adapted and thrived in pandemic-produced settings (Fleming 2020).

One small way we can give control back to students is to help them to explore the data and mathematics that are informing decisions affecting their lives. Models, graphs, and charts are used extensively by news and official agencies because mathematics helps us understand, interpret, and respond to new situations. Mathematics helps institutions, school districts, and government agencies fight COVID-19 because it enables us to predict potential outcomes of different policies and weigh the risks and benefits of a

large-scale problem. Furthermore, the mathematical skills involved in analyzing the COVID-19 pandemic are applicable to other problems in life.

Although mathematics is an important tool to understanding a pandemic, our research team found that many citizens have difficulty making sense of small percentages related to risks of COVID-19 (Yoon et al. 2021). This finding is consistent with other research that shows citizens struggle with percentages, decimals, and distinguishing between small numbers with different orders of magnitude (Carpenter et al. 1983; Mullis et al. 1991). Thus, making sense of such information as “a risk of death is 0.7 percent” or “a risk of hospitalization is 3 percent” can be difficult because most people rarely make ordinary decisions based on numerical quantifications of risk (Konold 1989; Reyna 2004; Stone, Yates,

and Parker 1997). During COVID-19, many citizens are interested in comparing COVID-19 to more familiar risks like influenza, but they find this mathematically challenging to do without support (Yoon et al. 2021).

To facilitate people using mathematical reasoning to compare risks associated with the COVID-19 pandemic, the National Science Foundation (NSF) funded the COViD-TASER (Creation of Visualizations of Data: The Application of STEM Education Research) research team to create the COVID-19 Relative Risk Tool (RRT) (see video 1 [link online] and figure 1). All the co-authors of this article are members of COViD-TASER. The RRT uses an interactive bar chart to display COVID-19 infection and vaccination risks alongside more familiar risks like driving, contracting breast cancer, playing soccer, and skydiving.

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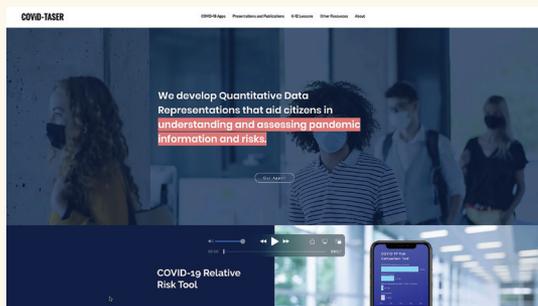
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To involve students, we developed and tested elementary, middle, and high school lesson plans that use the RRT. Each lesson is aligned with the Common Core State Standards for Mathematics (CCSSM; NGA Center

and CCSSO 2010) for both content and mathematical practice, and each plan pursues two goals: (1) making comparisons of relative size of quantities, and (2) interpreting these relative sizes to understand COVID-19 data.

Video 1 An Introduction to the COVID-19 Relative Risk Tool (RRT)

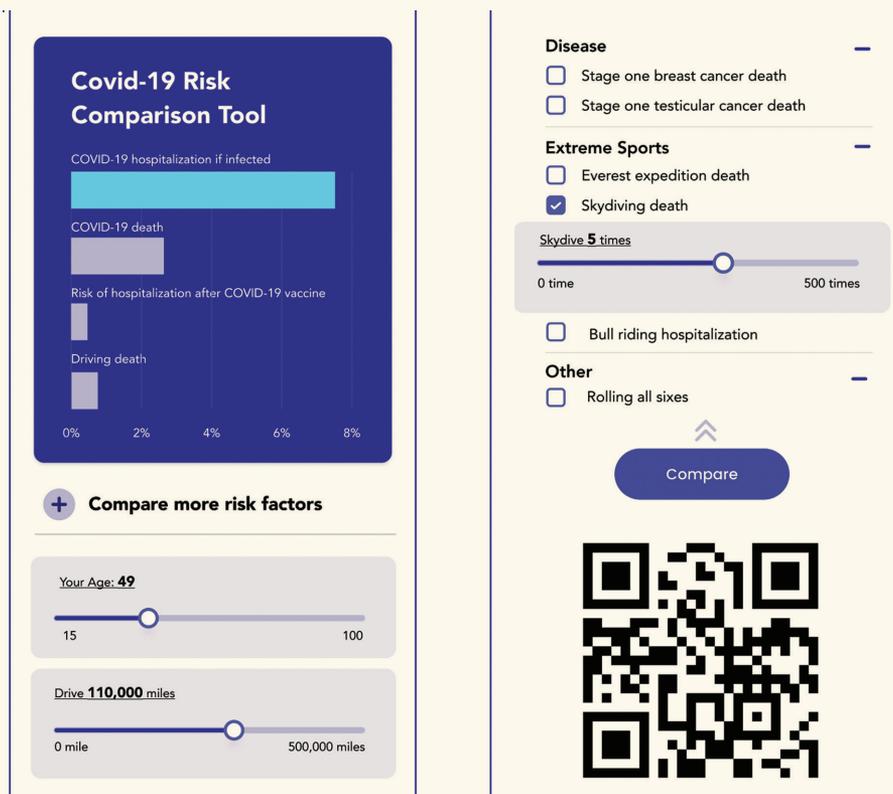


[Watch the full video online.](#)

FOCUSING ON RELATIVE SIZE

To compare the relative size of two quantities is to determine how many times as large one quantity is as another. This concept is embedded throughout curriculum in the United States beginning in the first grade when students measure one length with another length. In fact, any measurement, in any unit, is a relative size (Moore 2013; Thompson and Saldanha 2003). For example, to say that my stick is 4.7 feet means that the stick is 4.7 times as long as the length of 1 standard foot unit. CCSSM asks that students understand fractions as comparisons of relative size (Faulker 2013). This meaning for fractions helps them understand proportional relationships, rates, slope, scale factors, and derivatives (Byerley 2019; Thompson and Thompson 1996).

Fig. 1



The COVID-19 Relative Risk Tool (RRT) looks like this when displayed on a mobile phone. This QR code will take you directly to the RRT.

Making comparisons of relative size is productive when comparing risks and making decisions. For example, an April 2020 estimate of the death rates for influenza was 0.1 percent and for COVID-19 was 2.1 percent (Yoon et al. 2021). A person who can compare these rates only additively might conclude that COVID-19 is 2 percent more deadly than influenza—but to interpret this comparison still requires a sense of whether a 2 percent additive death risk increase is significant! People who can also compare these rates multiplicatively can conclude that COVID-19 is 21 times as deadly as influenza, thus allowing them to reach a more meaningful comparison: that however deadly influenza is (a risk people already knew about and had made decisions about), COVID-19 is 21 times as deadly as that known risk.

Research shows that although most students develop strong additive reasoning, students face much more difficulty developing multiplicative reasoning (Steffe and Olive 2009). COVID-TASER's research has illustrated the consequences of this for how individuals interpret COVID-19 data. For example, COVID-TASER's early research (Yoon et al. 2021) showed that, when asked to compare April 2020 estimates of the death rates for influenza (0.1 percent) and COVID-19 (2.1 percent), only 13 of 32 adults concluded that COVID-19 was 21 times as deadly as influenza.

The COVID-TASER team deliberately created the RRT (see video 1 [link online], figure 1) as a low-floor, high-ceiling environment to encourage people to use relative size to compare risks. Users can visually compare bar lengths to estimate that Event A is m times as risky as Event B and consequently bypass the difficulties associated with percentages, decimals, and division. The RRT design was based on research on student thinking to be comprehensible to the majority of citizens, including those who primarily reason additively and those who are comfortable coordinating only a small number of quantities at one time (Ulrich 2015).

Yet the RRT also allows users to identify the exact percentages, make precise calculations of relative size, and interpret their results to gain further understanding of COVID-19 and other activities' riskiness (as in our middle and high school lessons). The range of reasoning promoted and afforded by the RRT makes it suitable for use by elementary school students through adulthood; everyone can engage with the RRT.

We have labeled our lessons with grade bands, but each lesson can be used as a warm-up to later lessons. In our classroom trials, we discovered that all the lessons are accessible to all grades with appropriate teacher

support. Videos 2, 3, and 4 (links online) explain each lesson plan in detail and display the lesson slides, teacher notes, and handouts. All lessons are centered on goals (1) and (2) today. We find it important to note that the phrase *times as large as* is not as common in school curricula as we would wish, which is why we take the time to make that idea clear at the beginning of each lesson.

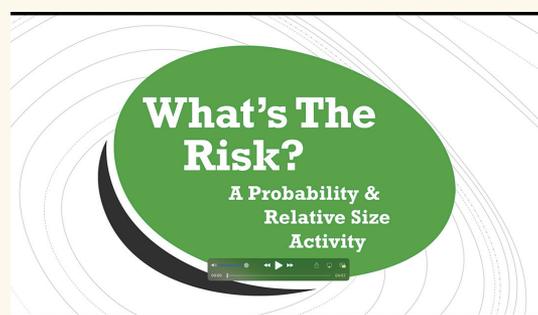
We next share our experiences carrying out these lessons in elementary, middle, and high school classes. The slide decks for each lesson are in the supplemental online material (link online). Please note that because videos 2, 3, and 4 (links online) cover the key mathematical ideas of each lesson, suggestions on implementation, and common misconceptions, watching the videos will be useful before reading the rest of this article. Each lesson is designed to be completed in a 50–60-minute class

Video 2 Elementary School Lesson Resources with Explanations



[Watch the full video online.](#)

Video 3 Middle School Lesson Resources with Explanations



[Watch the full video online.](#)

session, although it may take longer if a teacher uses large parts of earlier lessons as a warm-up.

Our lessons are aligned with multiple Mathematics Teaching Practices (NCTM 2014, p. 10). The real-life context of these lessons, as well as questions that ask students to apply their mathematical knowledge to making decisions in their own lives, aligns with the practice “implement tasks that promote reasoning and problem-solving.” By asking students to connect the bars’ lengths and associated decimals, teachers are asking them to “use and connect mathematical representations.” Implementing tasks in which students must substantiate claims using data from the RRT and multiplicative comparisons allows teachers to “facilitate meaningful mathematical discourse.”

- Elementary school example, go to the next section.
- Middle school example, go to page 344.
- High school example, go to page 346.

Video 4 High School Lesson Resources with Explanations



 Watch the full video online.

CARRYING OUT THE ELEMENTARY SCHOOL LESSON

Ms. Kite implemented the elementary lesson plan with her fourth-grade class. The lesson began with a conversation about the meanings of the words *risk* and *risky*. Most students thought risky is synonymous with *dangerous*. So when asked, “Should we ever do things that have some risk?”, the majority of the class said no. One student, however, said that sometimes taking a risk was a good thing and explained that it would be good for a parent to risk their own life if their child was in danger. Ms. Kite went on to explain that often, when you do something risky, it *can* be dangerous but not always.

Risk was also discussed in connection to COVID-19 and the pandemic. Ms. Kite thought this helped reinforce a message that she had told her students all year: COVID-19 is dangerous! She also found it important to have such a conversation because it gave students further vocabulary to articulate the concerns surrounding COVID-19, as well as ideas about risk in general (see figure 2).

Following this important opening conversation, the elementary lesson asks students to use their fingers to measure a larger bar with a smaller bar; students are shown how to repeatedly copy the smaller bar and place the copies end-to-end until their combined length matches the larger bar (see video 2 [link online]). They can then use their result of this iterative process (Tzur and Hunt 2015) to make a multiplicative comparison of the two lengths. The students describe this as a times-as-much comparison, such as “long-term COVID-19 symptoms are five times as likely as rolling three sixes, and rolling three sixes is one-fifth times as likely as long-term COVID-19 symptoms.” Students then practice making multiplicative comparisons and discuss what these comparisons help them understand about risk.

Fig. 2

A photograph of a student's handwritten reflection on a piece of paper. The text reads: "What have you learned about comparing risks? Like skydive 500 times or climb Everest once, even if it seems like skydive 500 times is more dangerous than climb Everest, the Everest is more dangerous. Did this activity change your mind about the riskiness of anything? I did know that you have to risk your life, but these changed my mind because you have to compare what's more risky or dying."

By the end of the lesson, students like Sierra began to realize the benefit of mathematically comparing risks of different activities. She shared how one of the comparisons in the lesson surprised her and made her reevaluate the riskiness of skydiving.

In Ms. Kite's class, she observed that students were first confused when introduced to the idea of a times-as-much comparison. However, as students continued through the lesson and made times-as-much comparisons, they began to understand the quantitative significance of the phrase (see figure 3). Ms. Kite's experience shows that a single lesson can help students begin to develop the idea of multiplicative comparisons (or relative size). But this idea takes time to develop, so our lesson was designed to prompt students to think about relative size across repeated opportunities.

Ms. Kite shared that for the students the relationship between the mathematical times-as-much comparisons and the COVID-19 context was unclear. Her students isolated the different parts of the lesson by not transferring their new knowledge of times-as-much comparisons to the data in the RRT. She pointed out that it was important that teachers stress the relationship between the current health crisis and the mathematics people can use to understand it. We modified the lesson after Ms. Kite's feedback, and we encourage teachers to focus on the reflection question after each set of mathematical comparisons. These questions include "Did this comparison surprise you?" or "How does this comparison affect your thoughts on COVID-19?" and help students tie their mathematical work back to COVID-19.

- Middle school example, go to the next section.
- High school example, go to page 346.
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CARRYING OUT THE MIDDLE SCHOOL LESSON

We tested our middle school plan in four middle school classes and here describe Dr. Joshua's class. Dr. Joshua,

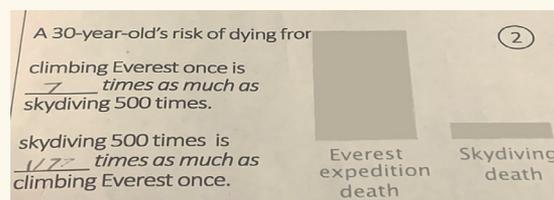
was teaching in-person/virtual hybrid middle and high school mathematics in spring 2021. Dr. Joshua's algebra 1 class was a small pull-out section for gifted students who have special needs such as being autistic, having attention-deficit/hyperactivity disorder, or having severe anxiety. Students, both in person and virtually, worked in pairs and showed their work on either physical whiteboards or Google Jamboard® slides.

Just as in the elementary lesson, Dr. Joshua asked the students to share their meanings for risk, and how people should decide if and when to take risks. Students agreed that risk was inevitable because "everything in life has risk" and then had a lively discussion about what constituted "too much risk." The prevailing opinion, eventually adopted by most students, suggested that a person should take any risk with a 49 percent or less chance of a negative outcome (see figure 4).

Dr. Joshua found this answer surprising because nothing that humans regularly engage in has a 50 percent chance of injury or death. After reading Konold's (1989) work, she realized that her students' answer suggested they had an *outcome approach* meaning. An outcome approach meaning involves interpreting a probability in terms of predicting the outcome of a single next trial (Konold 1989). So, a student with this thinking might interpret a 40 percent risk of death from COVID-19 as meaning that a single person who gets COVID-19 is probably not going to die, so that person does not need to deem COVID-19 as risky.

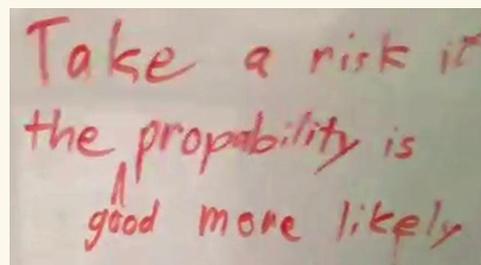
This exchange encouraged Dr. Joshua to continue with the RRT lessons and help students make multiplicative comparisons and understand *times-as-much-as* comparisons in a context that was central to their lives. Her experience also reinforced how important the

Fig. 3



Layla used her fingers to measure the size of the skydiving death bar and then iterated that size until she reached the size of the Everest death bar. She needed seven copies, so she concluded that climbing Everest was seven times as risky as skydiving.

Fig. 4



Theo's answer about risk, first written and then clarified verbally, suggested any risk under 50 percent was acceptable.

opening questions for the middle school lesson were. Dr. Joshua reflected that without these early discussions about the meaning of probability and percentages, her students would have been less prepared to interpret their risk comparisons. Additionally, she would have missed important information about her students' background knowledge that she used to guide the successive conversations.

Next, Dr. Joshua used the slides that asked students to reflect on their meanings for a 1 percent risk (see figure 5). Almost all the students gave an answer that implied probabilities can predict exactly what will happen with a small sample, such as 100 people. This meaning is much more productive than the outcome approach previously described, because people reasoning in this way can reach accurate conclusions about approximately how many deaths would occur in large samples such as a country or world population.

To further push the students on their meanings, we asked the students the perturbing questions in figure 6. A student, Ty, proposed an answer for 1 percent risk that

reflected a meaning for percent probability or risk as a “long-term relative frequency,” or the idea that as one considers more data, the observed ratio will generally get closer to the theoretical ratio (see figure 6). Upon considering Ty’s answer, the other students agreed with it and the class was able to discuss the nuances of how to interpret “1 out of 100.”

The question “Use the tool to fill in the blank: ___ is riskier than ___” was intended to encourage students to explore the RRT and find unique comparisons that interested them. Unfortunately, its implementation did not go as intended in Dr. Joshua’s class. Students wrote, “Getting COVID is riskier than not getting COVID,” and other simple statements. Dr. Joshua decided that in the future, she would encourage students to compete to come up with the most interesting or startling comparison using the RRT.

Once Dr. Joshua introduced them to making additive and multiplicative comparisons of specific activities, the students became more engaged. As Dr. Joshua expected, all the students quickly concluded that subtraction was the appropriate operation for making a “more than” comparison. However, several students did not realize that division was the appropriate operation for making a times-as-much-as comparison. The students who made this conclusion helped those who did not, and they quickly started through the last few slides. Most of the students chose the times-as-much-as comparison as more useful with every pair of quantities (see figure 7). Only one student chose the “more than” comparison as more useful because he said that the large value of the multiplicative comparison was hard to wrap his head around (see figure 8).

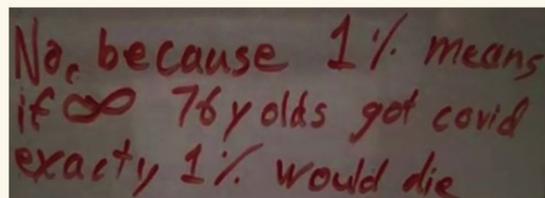
Fig. 5



Kay, like several of her classmates, initially gave a meaning for a 1 percent risk that implied it could completely predict what would happen with a small sample.

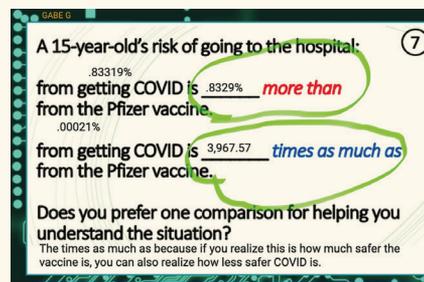
Fig. 6

- If I put 100 people aged 87 with COVID-19 in a room, will 1 of them definitely die?
- If I put 100 people aged 87 with COVID-19 in a room, and 1 dies, will everyone else definitely recover?



Ty revised his explanation to reflect a meaning that was most useful for large samples.

Fig. 7



Ji made both kinds of comparisons and then concluded that the times-as-much-as comparison better illuminated the difference in risks. Out loud he said that the small decimal percentage was more difficult to interpret.

Throughout the hour, this lesson provoked conversation about the role that COVID-19 and COVID-19 vaccines were playing in the students' lives. For example, Ji shared that he had already asked to get the COVID-19 vaccine when the Federal Drug Administration dropped the minimum age to 12, but his parents felt the vaccines were too new.

When the lesson ended, Dr. Joshua found it interesting that students did not express a strong preference for either kind of measurement although they had consistently picked times-as-much-as within individual tasks. When she asked about this, the students answered that the better measurement depended on the context, which is a viable response and emphasizes the importance of conversations that balance mathematics and context. They did express genuine interest in the RRT, asked for the link to be posted on the class website, and said that they wished they had had it earlier in the year.

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CARRYING OUT THE HIGH SCHOOL LESSON

The high school lesson alters the lesson format by including sample social media posts inspired by our online interactions. These posts incorporate an aspect of personal argument and opinion not included in the elementary and middle school lessons. The lesson asks students to use the RRT to create possible responses to each post and encourages students to use the times-as-much-as comparison when responding to posts. Our hope in discussing social media posts was

to encourage students to engage in fact-checking posts they see in their personal social media viewing.

We tested our high school lesson in six high school classes and here describe what happened in one algebra 2 and one precalculus class of Dr. Joshua's (same co-author/teacher as before). Both classes progressed similarly, so we combine them into one narrative.

Dr. Joshua's classes had repeatedly talked about COVID-19 and its effect on their lives in the previous few months, so they immediately began exploring the RRT and making multiplicative comparisons. There were a few difficulties with the multiplicative comparisons that were similar to the middle schoolers' challenges. Next the students worked in pairs to respond to each of the sample social media posts. Student responses to "Sally's post" were especially interesting because of how students decided to scale risks with respect to time. Some students compared Sally's overall lifetime risk of driving with the risk of one COVID-19 infection and concluded that driving was more dangerous (although not 10 times as dangerous). These students brought other considerations like hospitalization risk into their answers to explain why Sally should still get the vaccine (see figure 9). Other students reasoned that COVID-19 had been around for only one year, so it should be compared to Sally's average yearly miles driven; and they all concluded COVID-19 was 5 to 10 times as risky as driving (see figure 10). No students chose to think about comparing a single COVID-19 infection to a single day's drive. This surprised Dr. Joshua because she thought Sally's phrase "driving to work each day" suggested such a comparison.

Students were engaged in making their arguments and comparing them with one another. One class responded to three scenarios and the other responded to two. "Luisa's post" allowed students to respond in a variety of ways. Some focused on a specific vaccine like the J and J, which had been in the news recently (see figure 11), or on a specific age, assuming Luisa was their peer. Others used the sliders to scan through a wide range of ages before writing their response (see figure 12). Regardless of their approach, all students concluded that Luisa should still get the vaccine because COVID-19 was many orders of magnitude riskier than any vaccine.

At the end of each lesson, Dr. Joshua asked the students for feedback to gauge how useful the students found the RRT and lesson. Some students referred directly to the mathematics they practiced, such as "When you're evaluating risk, it's like a ratio and you can most effectively compare the two probabilities to each

Fig. 8

I prefer the "more than"; the "times as much" is too big a number for me to comprehend it or fully understand the implications

After making the same comparisons as Ji (see figure 7), Mickey explained that he preferred the additive comparison because the multiplicative comparison was too large to interpret.

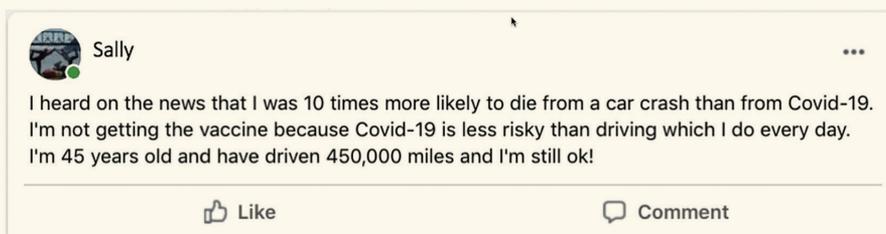
other with a times-as-much-as,” and “It puts certain risks that you might not grasp otherwise into contexts of risks that you do know.” Other students discussed the utility of the RRT, saying, “I’m definitely going to use this [RRT] because I’m already having conversations with people about these things,” and one student suggested a modification he would like to see (to split hospitalization risk for vaccinated and unvaccinated people) that we have since implemented. Dr. Joshua wondered if the students were merely being complimentary since

she had introduced the RRT. However, after the weekend, students in both classes spontaneously asked her for “that website we used on Friday,” which led her to believe that they would go back to the RRT.

OTHER HIGH SCHOOL IMPLEMENTATIONS

We also carried out this lesson in four more classes. Ms. Lavendar, a co-author of this article who was substitute teaching in spring 2021, used the lesson in three

Fig. 9



a) Is Sally’s claim that “she is 10 times more likely to die from a car crash than from Covid-19” accurate? Explain.

No, the odds of a driving death is 0.549 and the odds of a COVID death are 0.12773, the odds are 4.298 times as likely to dies from a car crash than COVID

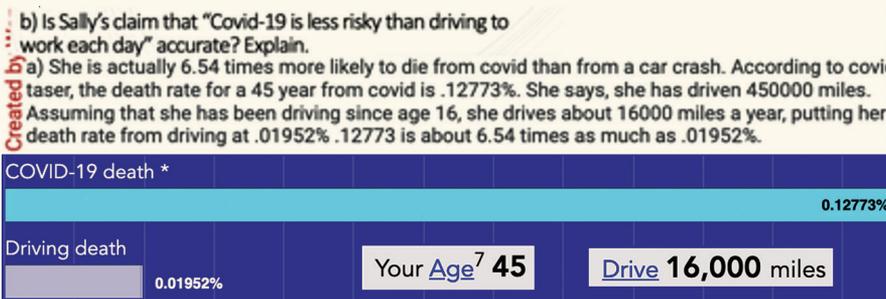
b) Is Sally’s claim that “Covid-19 is less risky than driving to work each day” accurate? Explain.

Not entirely, COVID could also lead to hospitalization which is far more likely than a driving accident (about 4%). If you get vaccinated then the odds of hospitalization is less than 1%.



Gabe explains his reasoning comparing COVID-19 risk to lifetime driving risk.

Fig. 10



Krista explains her reasoning comparing COVID-19 risk to yearly driving risk.

classes. She gave each student a different social media post, making sure that multiple students were assigned to each post but not sitting next to one another. After students had time to work with the RRT and reflect on their own, Ms. Lavendar brought the class together to go through the posts and listen to the students who had worked with that post. This allowed productive whole-class discussions to occur because students had their arguments already developed to share and contrast. Each discussion was led by two or three students who had thought about the post on their own, and then the rest of the class listened and contributed. Ms. Lavendar thought it was significant that she was a substitute teacher who had never met these students before (and the students had only returned to in-person school for a few weeks), but the students were engaged with both the mathematics and the practice of making data-based arguments throughout the lesson.

In contrast, Mr. Horne (a co-author of this article who was teaching high school mathematics in spring 2021) used the handout in his own class and allowed students to work at their own pace. However, this resulted in little whole-class conversation because each student mainly communicated with neighboring students. Many overarching ideas arose from the mathematics that Mr. Horne thought, upon reflection, would have been better used in a whole-class discussion. For example, some students discussed how COVID-19 is a repeated risk that comes simply from existing in public,

and we have to rely on the community around us to be protected; this is in contrast to risks such as skydiving or soccer injuries, which can be easily avoided purely through individual choices. Others responded that all decisions, including going to school or going out in public, are individual choices, and that all community mandates were inappropriate. Although these conversations may seem to veer away from the mathematics, these are exactly the nuances students need to explore in order to move from a mathematical calculation to using mathematics to make decisions.

Mr. Horne also shared that his students' exploration of the RRT made their discussions on COVID-19 richer because they were not resorting to parroting parents or media. The RRT gave them new information to use as they worked through their ideas with others. Mr. Horne also noted the conversation using the RRT was less polarized than other recent COVID-19 related student discussions. Using a common set of information contributed to more constructive dialogue. Mr. Horne expressed that if he were to teach the lesson again, he would periodically bring the class together to have these conversations because his first implementation failed to capitalize on all the potential of the lesson. He also decided that a productive follow-up would be to have the students draft their own social media prompt using data from the RRT to see how they benefited from the activity.

All three teachers concluded that the Relative Risk lesson was a good use of class time because students were engaging with real-world data, making arguments using the concept of relative size, and critiquing the reasoning of others using data.

Fig. 11

The risk of dying after taking the J & J vaccine is .00001%. However, the risk of Covid hospitalization unvaccinated is 4.0001% which is 400001 times as much as the risk from dying from the vaccine!

Stella focused on a controversial vaccine. Although she made a mistake (should be 400,010 times as much) her answer was close enough to reach an appropriate conclusion.

Fig. 12

For the average person, you are over 1,000 times more likely to die of COVID than the vaccine	Used age 45 - 1,200. Then looked at different ages
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Jay scanned through multiple ages to be able to make an age-independent conclusion.

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- Summary, see the next section.

SHOULD TEACHERS EVEN TALK ABOUT SUCH A SENSITIVE TOPIC?

As we implemented these lessons, a lot of opposing viewpoints about COVID-19 arose in discussion. We had a variety of comments expressed, including the following:

- We should wear masks—even though masks are uncomfortable—because of other vulnerable people.
- COVID-19 is really just a bad cold.
- Not enough people are taking COVID-19 seriously.

- Mandates should end because individuals should be responsible for their own risk.
- Too many people have died from COVID-19.
- Most COVID-19 deaths have been deliberately mislabeled by doctors.

Given the current political and social tensions in the United States, asking whether this activity and its content are appropriate for the mathematics classroom is a reasonable question. Because significant disagreements exist regarding the severity of COVID-19 and the subsequent governmental policies, is the topic really worth addressing? We argue it is for the following reasons.

First, as mathematics teachers, we regularly look for applications of mathematics that are real and relevant to our students. Nothing may be more relevant than a pandemic that has disrupted the life of every single person on the planet! By allowing students to compare COVID-19 risk to other normal activities and to critique the arguments of others with data-informed responses, we will help students realize that mathematics is important to understand their everyday lives.

Second, we believe topics that are high stakes, sensitive, and political are *especially* worthy of being addressed in the classroom. Being silent on such topics speaks volumes to our students. It implies that schools and mathematics classrooms are detached from the complexities of the real world and thus confirms to students that school is insignificant. Mathematics is not apolitical or acontextual; it is done by real people in real time with emotions and motives. Additionally, if the classroom cannot be a place where potentially divisive topics are respectfully discussed, what public forum is left?

Given the sensitivity of these topics, we want to acknowledge the friction teachers may experience in their classroom when discussing a politicized issue like COVID-19. The RRT can cultivate an initial conversation to discuss the mathematics of COVID-19. Furthermore, prior to and during the lesson, teachers should establish norms for the classroom conversation.

Confirmation bias is often difficult to overcome, and we saw this in the classroom. As we implemented these lessons in a total of 11 classrooms across all

grade bands, we found that students' responses did tend to follow their initial beliefs as shown in the warm-up. Students who thought COVID-19 was a serious community health problem focused on when the COVID-19-related risk was bigger, and those who thought COVID-19 was less serious focused on when the non-COVID-19 risk was bigger. However, we did see students making accurate comparisons of risk and dealing with the results of those comparisons, whether they verified the students' beliefs or not. The students had to wrestle with a common set of data that sometimes supported and sometimes contradicted their prior beliefs.

FINAL THOUGHTS

The RRT and its associated lesson plans give students a chance to use mathematics to analyze a pandemic that has affected many aspects of their lives. Our classroom experiences told us that the lessons succeeded in fulfilling our two goals for students: to practice making comparisons of relative size of quantities, and to practice interpreting these relative sizes to understand real-world data about COVID-19. Moreover, each of those goals was supported and justified by the other in an authentic way. The comparisons we asked students to make were not just for the sake of classroom learning—they are the same comparisons we hope all citizens (the original audience for the RRT) will make as part of everyday decisions in their lives.

For teachers who wish to incorporate more real-world contexts in the mathematics classroom and are unsure of how to start, we hope that our structured, pretested lessons plans provide a starting place for you to help your students make connections between the classroom and their everyday lives. Real-world lessons, in which the mathematics is necessitated by the context, can also encourage students to look at other societal issues and ask how mathematics might help in resolving them (Stephan et al. 2021). They can also inspire teachers to start developing their own ideas around bridging the divide between students' lives inside and outside the classroom. If we can have meaningful conversations about mathematics and COVID-19, in what other contexts can we find engaging mathematical discussions? —

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