Documenting the Emergence of "Speaking with Meaning" as a Sociomathematical Norm in Professional Learning Community Discourse

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A R T I C L E   I N F O

A B S T R A C T

We introduce the sociomathematical norm of *speaking with meaning* and describe its emergence in a professional learning community (PLC) of secondary mathematics and science teachers. We use *speaking with meaning* to reference specific attributes of individual communication that have been revealed to improve the quality of discourse among individuals engaged in discourse in a PLC. An individual who is *speaking with meaning* provides conceptually based descriptions when communicating with others about solution approaches. The quantities and relationships between quantities in the problem context are described rather than only stating procedures or numerical calculations used to obtain an answer to a problem. Solution approaches are justified with logical and coherent arguments that have a conceptual rather than procedural basis. The data for this research was collected during a year-long study that investigated a PLC whose members were secondary mathematics and science teachers. Analysis of the data revealed that after one semester of participating in a PLC where *speaking with meaning* was emphasized, the PLC members began to establish their own criteria for an acceptable mathematical argument and what constituted *speaking with meaning*. The group also emerged with common expectations that answers be accompanied by explanations and mathematical operations be explained conceptually (not just procedurally). The course and PLC design that supported the emergence of *speaking with meaning* by individuals participating in a PLC are described.

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1. Introduction

There is concern in the United States about the performance of the nation's children in mathematics and science. Due to this concern, science and mathematics faculty from institutions of higher education are partnering with K-12 districts to form a mathematics and science partnership program (http://hub.mspnet.org/index.cfm/home). The primary goal of this organization is to improve math and science education via five primary features: Partnership-Driven, Teacher Quality, Quantity and Diversity, Challenging Courses and Curricula, Evidence-Based Design, and Institutional Change and Sustainability. The research presented here is situated within a larger project that attempts to implement these features in five school districts. While we believe each of these key features is important, the research in this paper reports results from our investigations

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to improve teacher quality. The broader research project, which this study is a part of, intends to impact the participants' teaching by improving both their conceptual and pedagogical content knowledge and by helping them learn to be reflective about their instructional practices and its impact on student learning. An assumption of this project was that strong conceptual and pedagogical content knowledge is needed for improving teaching practice. Specifically, we studied the discourse of teachers as they worked together in professional learning communities (PLCs) that were designed to support teachers in reflecting on the learning and teaching of mathematics concepts.

Our study of the communication patterns of individuals in four PLCs during an academic year revealed that individual PLC members failed to communicate meanings to one another when engaged in discourse. Members of the PLCs were often focused on performing procedures and their explanations often focused on these procedures as opposed to the underlying concepts of the problem. We observed that conversations between PLC members did not result in an exchange of ideas and the teachers were observed talking past each other with little discussion of student learning and what is involved in understanding mathematics concepts. These findings prompted us to introduce the term *speaking with meaning* to the teachers in the project. *Speaking with meaning* refers to specific attributes of communication that we wanted to promote in individuals when they engaged in discourse with their colleagues during the PLC meetings and course. We use the term *speaking with meaning* to convey that responses are conceptually based, conclusions are supported by a mathematical argument that can be understood by the intended audience, and explanations, when appropriate, refer to actual quantities and how they are related. This paper describes our attempt to promote *speaking with meaning* by individual teachers within a specific PLC in our project. We also describe the emergence of *speaking with meaning* as a sociomathematical norm in one professional learning community of secondary math and science teachers.

2. Theoretical perspective

Thompson, Philipp, Thompson, and Boyd (1994) identify differences between teachers and students with calculational and conceptual orientations. According to the authors, individuals with a conceptual orientation have a rich conception of situations, ideas, and relationships among ideas, while individuals with a calculational orientation are more concerned with the steps needed to obtain a solution. An individual with a calculational orientation typically places an emphasis on performing procedures and has a tendency to provide a solution consisting solely of numerical operations regardless of the context. When probed to provide a rationale for a solution the individual typically responds by describing operations and calculations, with no explanation of why a particular approach was selected. In contrast, an individual who has a conceptual orientation is more concerned with the overall context in which the problem is situated. Explanations typically reveal the presence of a broader system of ideas and ways of thinking. Justifications also typically include reference to the problem's context and are such that the conceptions behind the solution are explicit to the entire audience.

Speaking with meaning draws heavily from the notion of a conceptual orientation, which places a strong emphasis on attempting to communicate meaningfully with others. We use *speaking with meaning* to describe the type of meaningful discourse that is expected when individuals are involved in explaining their solution to a problem, discussing others' solutions to problems, and/or speaking about learning or teaching an idea.[1] *Speaking with meaning* implies that responses are conceptually based, conclusions are supported by a mathematical argument, and explanations include reference to the quantities in the problem context. In contrast, the absence of *speaking with meaning* implies that the explanations of the solution approach are not grounded in the context of the problem; rather, the focus is on describing the procedures and calculations used to determine the answer. Because of this distinction, an absence of *speaking with meaning* can occur in both mathematically correct and incorrect responses.

To briefly illustrate *speaking with meaning*, consider the problem in which a photo of width 8 in. and height 10 in. is enlarged to a width of 32 in. A student is asked to find the height of the enlarged photo. In this interaction the student provides an answer with a calculational orientation and the teacher accepts this explanation.

Student 1:   The answer is 40.
Teacher 1:   How did you get that?
Student 1:   Well, 32 divided by 8 is 4 and 4 times 10 is 40.
Teacher 1:   OK, good.

An interaction in which the student provides a more conceptually oriented response follows.

Student 2:   The answer is 40.
Teacher 2:   How did you get that?
Student 2:   Because of the relationship of height and width. . .if the width is increased by 4 times, the height would be increased by 4
               times as well. Since the ratio of width to height in the original photo is 4 to 5, then if the width is increased by a factor of 4, the
               length must be increased by a factor of 4. This will assure that when I set up the ratio of the new photo, the ratio of width to
               length will still be 4 to 5.
Teacher 2:   OK, good.

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[1] We note here that what emerges as the criteria of *speaking with meaning* within a PLC or a classroom will differ from setting to setting.

The explanation given by Student 1 describes the calculations used to determine the answer with no mention of the quantities of length and width and how they are related. This calculational focused response does not provide insight into the thinking that supported the approach. His response revealed that he was able to manipulate the given numbers to get the correct numerical answer; however his response could not be characterized as one in which he *spoke with meaning*. In contrast, the explanation given by Student 2 focused on the quantities of the situation and included discussion of the proportional relationship between the width and height of the photo.

As previously described, Thompson et al.'s notion of conceptual orientation initially informed our construct of *speaking with meaning*. However, in our attempt to promote *speaking with meaning* in the PLCs and the graduate course (described in more detail in the next section) in which the teachers were concurrently enrolled, our research team needed to determine what *speaking with meaning* would entail for each concept taught in the course. Our criteria for *speaking with meaning* relative to these specific ideas were constructed from the understandings and discourse that we believed to be valued by the mathematical community. As an example, Carlson (1998) and Oehrtman, Carlson, and Thompson (2008) describe the importance of the function concept in understanding key ideas of precalculus and calculus and the authors describe the ways of thinking and understandings that are needed for using and understanding the function concept (e.g., dynamic conceptualizations of two quantities, covariational reasoning, process view of functions, etc.). In light of these central understandings, the authors provide suggestions for promoting a process view of functions (a view of function that involves a dynamic transformation of quantities) and also illustrate how covariational reasoning (attending to how two quantities change in tandem) builds from a process view of function. The authors describe the thinking and speaking of someone who is using covariational reasoning to analyze dynamic function events. They suggest that in order to promote a student's ability to engage in covariational reasoning, the student should be prompted to explain how input quantities are transformed by the function to produce output quantities. They also describe that speaking about the actual quantities and how they are changing together is critical for expressing the covariational relationship conveyed by a function's graph.

Research that has revealed the ways of thinking and understandings needed for meaningful use of a concept has informed *our* criteria for *speaking with meaning*.[2] Because of past research that has revealed the complexities involved in understanding key ideas deeply, we expected that the teachers in our project would initially have difficulty *speaking with meaning*. We conjectured that they would initially lack necessary understandings and connections to *speak with meaning* in the course and PLC sessions. However, we did expect that our interventions would result in the teachers acquiring understandings and reasoning abilities that promoted *speak with meaning* as the semester progressed.

Social aspects of the emergent perspective, social norms, and sociomathematical norms also informed this research. Social norms are implicit understandings about participation. Sociomathematical norms refer to normative behaviors that are specific to mathematics, such as understanding what constitutes an acceptable mathematical solution, and emerge from what counts as acceptable mathematical behavior in the classroom (Yackel & Cobb, 1996). One example of a social norm is that an individual should justify her/his solutions, while an example of a sociomathematical norm is the establishing of criteria for what counts as a sufficient justification for a solution. We use the sociomathematical norm of *speaking with meaning* to illustrate the type of mathematical behavior we expected of the teachers as they participated in their PLCs and the course.

We conjectured that through the expectation that participants of a class or PLC *speak with meaning*, the discourse of the participants would progress towards being conceptually oriented. This progression was conjectured to occur as norms such as what is considered a sufficient explanation and what counts as a mathematically different solution were also negotiated. Yackel and Cobb (1996) claim that these sociomathematical norms can support higher-level cognitive activity. Thus, the promotion of *speaking with meaning* can influence the constitution of these norms in ways that foster higher-level cognitive activity and more meaningful discourse.

As the expectation of *speaking with meaning* is initiated and supported, explanations themselves can become explicit objects of discourse and reflection, where individuals are able to facilitate discussions themselves and make judgments about what others say. In order for an individual to make these judgments and consider how not only themselves, but also others make sense of explanations, an individual must shift from only giving or listening to explanations to making the explanations themselves objects of reflection (Yackel & Cobb, 1996). In the reflection of explanations (both one's own and others), individuals have the opportunity to build understandings and make connections. In addition, if the group is actively modeling *speaking with meaning*, explanations are conceptually based and promote deeper understandings and stronger cognitive connections.

Our observations of PLC meetings also revealed that the specific phrase "*speaking with meaning*" was a tool that the facilitators used to emphasize the type of discourse they desired by individuals in their PLC meetings. The facilitators were observed requesting that a member of the PLC "*speak with meaning*" when the facilitator believed that a PLC member's responses or explanations were not sufficient. Over time, other members of the PLC also began to use the phrase during communication with their colleagues. For instance, members of the PLC would explicitly challenge one another to "*speak with meaning*." As the members began challenging each other to "*speak with meaning*," we observed that the communication between individuals in the PLC was much more meaningful and productive.

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[2] We note that acquiring these understandings and ways of thinking is non-trivial. Carlson, Jacobs, Coe, Larsen, and Hsu (2002) research reveals that traditional curriculum has not promoted conceptual understanding of the function concept in high performing undergraduate students.

These observations have resulted in our view of *speaking with meaning* as having a dual nature: as a sociomathematical norm and as a tool for raising awareness of desirable communication patterns for which individual teachers should aspire. Examples of communication patterns are provided to illustrate the usefulness of *speaking with meaning* and how this standard for communication emerged in a PLC. We have also described how members of a PLC used the phrase *speaking with meaning* to convey standards for communication by individual teachers within a PLC and students in a classroom. The progressive and eventual spontaneous use of *speaking with meaning* by the teachers revealed that they found the phrase to be useful tool for raising the standards for communication in their classrooms and the PLC sessions. It was by focusing on the meanings conveyed by the individual that the quality of discourse of a PLC or class was improved.

3. The setting for the study

Project Pathways is a 5-year project in which STEM scientists and educational researchers are partnering with four secondary school districts to produce a research-based and tested model for enhancing instruction of mathematics and science in grades 9–12. This Math and Science Partnership (MSP) is investigating the effectiveness of a four course graduate sequence with accompanying professional learning communities. In partnership with school leaders, university faculty developed four mathematics/science graduate courses, all of which had a focus on modeling and a connecting theme of covariation, i.e., models emerged from attending to the variation of two quantities as they changed in tandem (Carlson, 1998; Carlson et al., 2002). The university faculty intended these courses to support secondary mathematics teachers in developing knowledge of foundational concepts of introductory math, science and engineering courses at the university level. The secondary mathematics and science teachers that are the focus of the MSP study attended the course one night per week and met for 1 h each week in PLCs. The primary purpose of the PLC meetings was to support teachers in engaging in meaningful discussions about issues of learning and teaching concepts that were central to the course. The courses and accompanying 1-h PLC meetings were held at school sites. The data for this study was collected in the first of the four-course sequence and during the accompanying PLC meetings.

3.1. Promoting speaking with meaning in the course and PLC's

A university faculty member instructed the graduate course and made explicit efforts to promote meaningful discourse in the class by assuring that individual teachers *spoke with meaning* when communicating to the class and during small group discussions. On the first day of class the instructor led a discussion about the role of communication in learning and using mathematics concepts to solve problems. This discussion led to the negotiation of specific goals for communication during the class and in the PLC sessions. The instructor prompted the teachers to describe how they could individually contribute to a classroom culture in which the communication was meaningful. The teachers volunteered responses indicating that they would need to: (i) listen to and attempt to make sense of the meanings conveyed by the teacher and their colleagues; (ii) attempt to communicate meanings when they spoke; (iii) respect the learning process of the individual; and (iv) ask for clarification if someone else's meanings are not clear. The class decided to call these explicit communication goals "Rules of Engagement." The explicit negotiation of these goals appeared to contribute to most teachers valuing them as something worthy of striving for when communicating during the class and PLC sessions.

During the first class session the instructor posed mathematical questions, consistently listened to the meanings conveyed by the teachers, and made specific prompts to support the teachers in *speaking with meaning* when providing their responses. The instructor also attempted to model *speaking with meaning* whenever she spoke. Throughout the course the instructor frequently prompted the teachers to provide a rational for a solution approach. She also made regular prompts for the teachers to reference the quantities and relationships in a problem context. She further encouraged them to avoid use of pronouns when describing a solution (e.g., instead of saying "it is growing," say, "the amount of money in the account is increasing as the number of years since 2000 increases). Also, in an effort to be explicit about the type of communication expected, the instructor publicly called attention to instances when a teacher *spoke with meaning* during the class. This move by the instructor is an example of the use of the phrase "*speaking with meaning*" as a tool that an instructor can use in a classroom.

The PLC facilitators were expected to promote *speaking with meaning* during the PLC sessions. Initial investigations revealed that the facilitators had difficulty modeling and reinforcing these behaviors. It appeared that their mathematical understandings at this stage of the course were too weak to support meaningful communication.[3] They also appeared to have little experience reflecting on or attending to the meaningfulness of their speaking. This led to our developing facilitator workshops designed to support the facilitators in modeling *speaking with meaning* and promoting meaningful communication among members of a PLC. During the workshops the teachers viewed videos of their PLC sessions for the goal of helping the teachers recognize how and when to intervene during discussions to assure that the PLC members were engaged in meaningful communication. The workshop leaders used videos of PLC sessions to illustrate communication patterns that were meaningful and ones that were not meaningful. After showing a particular video clip, the workshop participants were prompted to identify instances when the PLC facilitator promoted *speaking with meaning* and when the facilitator missed

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[3] Mathematical understandings are critical to an individual's ability to speak with meaning; however our current research suggests that although understanding a concept is necessary for speaking with meaning about a concept, it is not sufficient.

opportunities to support *speaking with meaning*. The PLC workshop leader also held a 30-min coaching session (described in more detail in the next section) with the PLC facilitators after each class session.

3.2. How do we define PLCs?

A PLC is a collection of math and science teachers from the same school (ideally) with one teacher designated as a peer facilitator. Our PLC design draws heavily from investigations of lesson study. We used a design research approach (Brown, 1992; Cobb, 2001; Edelson, 2002) in which iterations of instructional design for PLCs provided the setting for building our understanding of the PLC discourse in relation to the PLC facilitator actions during the PLC session. A primary purpose of the PLC sessions was to engage the PLC members in *meaningful discourse* about issues of knowing, learning and teaching secondary mathematics content that was related to what the PLC members were learning in the course. For the purpose of this research we describe "meaningful discourse" as communication about knowing, learning and teaching that draws on coherent understanding of the content and processes of learning the content. This definition of meaningful discourse is aligned with the framework upon which the course was designed and the understandings to be promoted within the course modules. As noted above, efforts to *speak with meaning* by individuals contributes to meaningful discourse among individuals. *Speaking with meaning* is more specific to the explanations of the individual group members, whereas meaningful discourse refers to a sequence of interactions of the group. The negotiation of what constitutes *speaking with meaning* by individuals within a group influences the overall discourse of the group and its "meaningfulness."

3.3. The PLC facilitator and PLC sessions

Each PLC had an assigned peer facilitator who was charged with managing the discussions during the PLC meetings. Our intention was that the facilitators guided the discussions by posing meaningful questions to promote quality discourse among the PLC members. The facilitators were selected based on their leadership abilities as recommended by their district math/science coordinator and were initially trained during three 6-h summer workshops. They were also supported in improving their facilitation ability during weekly coaching sessions. The workshops were designed to assist the facilitators in promoting quality discourse and *speaking with meaning* in their PLC. The workshop leaders engaged the PLC facilitators in a series of activities designed to improve the facilitators' abilities to both listen to the quality of mathematical explanations relative to their conceptual nature and learn to pose questions that promoted reflection on what is involved in understanding, learning, and teaching specific mathematical ideas (e.g., rate of change, proportions, and exponential growth). For instance, one activity included viewing and discussing videos of students as they explained their thinking when responding to mathematical tasks. In order to improve the facilitators' ability to attend to the thinking of others, the facilitators were asked to discuss what they could infer about student understandings from the video and to provide a rationale behind these inferences. In addition to the weekly meetings, the facilitators also attended three 3-h workshops during the semester. These workshops focused on specific attributes of facilitating that included their ability to ask questions to promote conceptual explanations and conversations.

A project leader led the weekly facilitator coaching sessions, where the facilitator coach met weekly for 30 min with the four facilitators from one course. During each coaching session, the facilitator coach asked questions about the facilitation behaviors exhibited by each facilitator during their previous PLC sessions. These questions were aimed to promote reflection about the quality of various PLC interactions that had been observed by the PLC coach after viewing videos of the PLC sessions from the prior week. As the coach viewed these videos with the research team before the coaching meetings, they discussed the facilitators' effectiveness in promoting and enacting *speaking with meaning* about issues of knowing, learning and teaching the content that was the focus of that PLC agenda. These discussions provided opportunities for the coach to address specific interactions during the coaching meetings that either promoted or inhibited meaningful discourse among members of a PLC.

The coach's strategies for promoting reflection about PLC interactions progressed from general discussions to making specific prompts to each PLC facilitator. During the first few coaching sessions, the facilitator coach discussed positive moves that she and the research team were noticing in hopes that the PLC facilitators who were less effective would begin to adopt the more effective strategies. As one example, she noted that when PLC members appeared to be speaking past each other, the facilitator of the PLC prompted the PLC members to put a written product on the whiteboard and encouraged the PLC members to speak about the ideas of the problem. The facilitator coach also gave general suggestions that she believed would promote positive facilitator moves, such as making an effort to listen to the meanings that the PLC members were communicating and trying to ask questions based on these meanings. As the PLC coach sensed that the facilitators were becoming more comfortable in their role as a facilitator, she became more direct with each of the PLC facilitators about behaviors that were less effective. For instance, the research team identified that some facilitators did not appear to probe members of their PLCs when weak or vague explanations were given. In the following coaching sessions, the facilitator coach reviewed videos with the facilitators and discussed opportunities when probing questions could have been used. As another example, after she viewed a facilitator that took the role of being the main contributor to the PLC sessions and not allowing members to discuss their own reasoning, she asked the facilitator why she never allowed a member to answer a question. The coach followed this by reminding the facilitator that all members should contribute to the discourse of the PLC equally and the focus should not be on the facilitator. The tools the coach provided in these previous examples were questioning strategies that would allow the members of the PLC to contribute in rich and meaningful ways.

The facilitators were also provided weekly PLC agendas that were created by university faculty and given to the facilitators prior to their PLC session. The PLC agendas included tasks designed to promote reflection and discussion among the PLC members about knowing, learning, and teaching concepts that were central to the course. As one example, the PLC members were asked to conduct an interview with a student for the purpose of gaining insights about what the student understood about an idea. The PLC agenda then prompted the facilitator to ask specific questions to the PLC members that would promote discussions about the central issues of learning that concept. Another example was the PLC agenda supporting the PLC members in developing, teaching, studying, and refining a lesson.

The PLC agendas also were intended to support the enactment of specific social norms for the PLC members' interactions. Such norms included both the expectation that teachers listen to and try to make sense of each other's meanings and solutions and that when discussing a solution or mathematical idea, they offer justifications for their claims and statements. This created an environment in which the PLC members negotiated what would constitute a meaningful mathematical argument, and what they deemed as *speaking with meaning* in their PLC.

In addition to assisting in the initiation of social norms for the PLC, the PLC agendas also attempted to promote *speaking with meaning* by explicitly reminding the facilitator to encourage the members of her/his PLC to *speak with meaning*. Thus, the facilitators used the term "*speaking with meaning*" to promote quality discourse among PLC members, where the members of the PLC negotiated the meaning of the phrase *speaking with meaning*. This resulted in *speaking with meaning* emerging as both a tool and a sociomathematical norm that was adapted and adopted by a PLC.

In this paper we focus primarily on the emergence of *speaking with meaning* as a sociomathematical norm in a PLC. As a result we elected to attend to the types of behaviors that were becoming normative among the members of the PLC. As the representatives of the mathematical community, it was the responsibility of university faculty members to provide the PLC facilitators with training so they would be aware of the nature of the discourse that was more desirable. The facilitators were supported in promoting quality discourse among the PLC members and within this context the PLC members' actions revealed what they believed were acceptable forms and patterns of communication.

4. Methods

4.1. The subjects and setting

This study investigated the communication patterns of PLC members from one PLC. All members of this PLC were con-currently enrolled in the first of a four-semester graduate course sequence designed for secondary mathematics and science teachers. The first course engaged teachers in activities that involved their: (i) collecting data; (ii) examining patterns in data; and (iii) developing mathematical formulas, graphs, and tables to represent and explain the data. The course focused on topics of rate of change, proportionality, polynomial functions, exponential growth and periodic behavior. The lessons were inquiry focused and the teachers were encouraged to engage in sense making as they worked in groups to complete mathematical tasks.

The PLC that was the focus of this study was composed of two secondary mathematics teachers and three secondary science teachers. The facilitator was a science teacher with less than 5 years of teaching experience. His performance in the course, as revealed in his homework and exams, supports that his understanding of the mathematical ideas central to the course was weak, although they did improve as the course progressed. It is also noteworthy that two members of this PLC had the strongest understandings initially; they also appeared to make positive shifts in their understandings over the course of the semester.

4.2. Data collection and analysis methods

Members of the research group videotaped all PLC meetings for the four PLC facilitators. These videos were then digitized and analyzed by the researchers who videotaped the meetings. Members of the research group coded the videos using Studiocode (2007) for various facilitator behaviors, and specifically for occurrences of (or lack of) *speaking with meaning*. The research team identified instances when the PLC members were engaged in mathematical tasks and when the PLC members made conjectures or statements that provided an opportunity for them to *speak with meaning*. This included opportunities to provide mathematical arguments or justifications for their statements and claims. Our analyses were focused on these arguments, and the group's reactions to these arguments.

Following the identification of these various instances, the research team of faculty and graduate students reviewed these episodes weekly. During these meetings, the episodes were discussed for the purpose of characterizing the quality of discourse exhibited by the PLC members. The initial analysis and viewing of PLC videos provided the initial documentation of *speaking with meaning*. As noted above these products also informed the facilitator coach during her weekly coaching sessions with the four PLC facilitators from the course.

Specific episodes that were identified to reveal a transition of what the PLC groups appeared to negotiate as *speaking with meaning* were transcribed for deeper analysis. This analysis involved looking for instances that demonstrated *speaking with meaning* as a normative behavior. To determine if *speaking with meaning* was becoming normative, the research group looked at instances when (i) individuals missed opportunities to speak *with meaning* and when (ii) individuals *spoke with meaning* without prompting. When individuals were not *speaking with meaning*, the research group noted the frequency

with which members challenged those who were not *speaking with meaning* and if this challenge resulted in the individual making adjustments in his speaking that were more indicative of *speaking with meaning*.

5. Results

We report data from analyzing the communication patterns among members of a PLC. Select transcripts that occurred during the semester demonstrate the emergence of *speaking with meaning* as a normative behavior as the PLC members met weekly during the 2006–2007 academic year.

Three manifestations of *speaking with meaning* were observed in our data.

1. Absence of *speaking with meaning.*
2. Participants should *speak with meaning* (social norm).
3. *Speaking with meaning* (sociomathematical norm).

Absence of *speaking with meaning* occurred when responses were not conceptually based or an explanation was given that was not grounded in the context of the problem (e.g., teachers who provided procedural explanations to a given problem). We have used the phrase "procedural explanation" to describe a response that focuses on the computational steps to getting an answer rather than the rationale for the solution approach. The explanation may also reference operations on numbers rather than relationships between quantities.

As the semester progressed, the social norm that participants should *speak with meaning* emerged. During PLC sessions, the teachers began attempting to justify their responses with more than just procedural explanations. In these cases, the members often used the context of the problem in their explanations and made observable attempts to provide a rationale for their approach; however, their explanations were often insufficient in that they did not include a coherent justification for their solution approach. We characterize the members' belief that participants should *speak with meaning* as a social norm that emerged within the PLCs.

In fostering the emergence of the sociomathematical norm for what would constitute a sufficient explanation or justification, the teachers began to hold each other accountable for providing more meaningful explanations, and hence establish criteria for what it meant to *speak with meaning*. Over the course of the semester, they began demanding that justifications be conceptual and that individuals include a clear articulation of the quantities and the relationships inherent in the problem under discussion. For example, explanations regarding an increasing rate needed to include a description of the changing relationship of the two quantities (e.g., how the amount of distance covered increased when considering equal amounts of change in time). Therefore, relative to *speaking with meaning* about rate-of-change, a criterion was established that the members were expected to speak about how the two quantities changed together (i.e., the covariation of the quantities).

We note that in discussing the three manifestations of *speaking with meaning*, and namely the last two manifestations, we do not intend these to be interpreted by the reader as a progression nor are they mutually exclusive. For instance, in order for the group to interactively determine what counts as *speaking with meaning*, the group members must initiate and sustain the social norm that participants should *speak with meaning* when discussing mathematical concepts.

These three manifestations of *speaking with meaning* emerged when analyzing the discourse patterns of the teachers when speaking with each other in their PLCs. It was only in watching videos of the PLC meetings that we observed the emergence of *speaking with meaning* as normative behavior. We noted that individual PLC members began to make explicit efforts to reference the quantities in a problem context and to provide a rational for a solution approach. They also began to spontaneously probe their peers to refine their communication so that it carried meaning to the listener. The results illustrate the emergence of *speaking with meaning* over one semester for the PLC of focus. We present three episodes occurring at three different times of the semester: the beginning, middle, and towards the end.

The following interaction occurred during the second session of the PLC facilitated by Jason. The focus of the discussion between the PLC members was the important ideas from that week's class session. The group initially discussed the notion of covariation and, at the onset of this discussion, Jason provided guidelines for the PLC discussions (*Excerpt 1*).

Excerpt 1.

1	Jason:	. . .specifics on to how to attack the problem or let's make sure we're
2		speaking meaningfully when we describe some of the big ideas of the
3		problems, not necessarily how we're gonna, how we're gonna teach it. But
4		how about the mathematical ideas? You remember anything that came up
5		on the mathematical part of the problem that we had on class? Any new
6		terms or new tools that we used?
7	Carroll:	Co. . . Co-variation?
8	Jason:	Yeah! Alright what did, do you remember what that? Maybe give me a
9		little discussion on that?
10	Carroll:	Uh, I think is kinda like what Andrew was talking about earlier just about
11		how when one, when one thing varies the other one has to vary in a
12		relationship and it can go either way it can go, um. . .proportionally or not
13		proportionally. [Laughs]
14	Richard:	No, I know what you're saying – kind of like the proportional, you know

15		when it's going straight down.
16		(short pause)
17	Richard:	You know, it's not proportional while it's not as blatantly obviously
18		proportional when it's doing the volume. [Motioning with hands/fingers]
19	Andrew:	It's not linear.
20	Richard:	Yeah.
21	Carroll:	When we did width versus height you know, you had to start up here and
22		work your way [Motions with left hand like it's up on the *y*-axis and *x*-axis
23		as if it's near or at the origin]. Okay, yeah I'm done. [Smiles]

During this exchange, the explanations given by the members of the PLC exhibited an absence of *speaking with meaning*. The facilitator's use of the phrase "*speaking with meaning*" appeared to be an attempt to initiate that the members should not just speak in terms of procedures, but also in terms of concepts (Lines 2–4). Judging by the ensuing conversation, the members did not appear to have a strong image of what it meant to *speak with meaning*. Members of the PLC, including the facilitator, appeared to be satisfied with a description of covariation as a proportion, although the members did not identify the quantities that are covarying, nor did they discuss how the quantities change in relation to each other. The members mentioned issues of proportionality and linearity but did not explain how these ideas related to the covariation of the situation. Although Carroll discussed two separate quantities when she said, "when one thing varies the other one has to vary in a relationship," she did not mention the specific quantities in the situation; nor did she mention how the two quantities may vary together (e.g., explaining what she meant by relationship) (Line 11). Her vague response did not convey an understanding of the meaning of covariation and no other member of the PLC questioned her explanation. Also, Andrew mentioned that "it's not linear," but the group did not bother to discuss how this pertained to their explanation of covariation (Line 19).

Andrew, Richard and Carroll each mentioned mathematical ideas (i.e., proportionality and linearity) that were related to covariation; however, they provided no explanation of how or why they thought the ideas were related to covariation. It is not clear why the members did not *speak with meaning* and in these types of situations it is the responsibility of the facilitator to ask for explanations from the other members that would reveal their understanding of proportionality and covariation. However, the absence of *speaking with meaning* during this interaction was not surprising as this session was early in the semester and in order for *speaking with meaning* to become normative, the members of the PLC must interactively constitute it. At this stage of the semester, it was likely that the PLC members did not have a clear image of what it meant to *speak with meaning* relative to that intended by the research team and the members likely had not had the opportunity to negotiate this meaning.

As the semester progressed, actions of the PLC members began suggesting that they believed that participants of the PLC should *speak with meaning*. The data also supported that they began to establish criteria for what constituted *speaking with meaning*. In this case, the criteria were that the members should refer to the quantities involved in the problem and justify their solutions with a mathematical argument. It was during the middle of the semester that it emerged as being normative that participants should *speak with meaning*.

The following excerpts (*Excerpts 2–4*) occurred mid-way through the semester and involved the PLC members discussing their previously determined solutions to the photo enlargement problem (Appendix A) in which a photograph with a 6 in. base and 8 in. height had to be enlarged so that the new base would be 15 in. The PLC members had already correctly determined that the height would be 20 in. and then discussed how they would explain this problem to their students (*Excerpt 2*).

Excerpt 2.

1	Jason:	What would you want your students to see if you know that there is a
2		. . . there are two quantities that are in a constant ratio? What would
3		you want your students to say or see?
4	Richard:	That . . . what I think is maybe that even though the uh difference in
5		numbers six to fifteen and eight to twenty and like the amount . . .
6		the total amount that they went is different . . . the uh I was gonna say
7		percentage wise they went up it's the same. Like they like . . . what I got
8		was uh . . . six was increased or . . . yeah six is forty percent of fifteen
9		and eight is forty percent of twenty. So they both . . . so the . . . so what I
10		got was the photo enlarged forty percent like even though they are
11		different numbers forty percent of a bigger is gonna be a bigger number
12		and forty percent of a smaller number is gonna be a smaller number.
13		So that even though the numbers between the difference between the
14		numbers . . . that the overall way they are changing I guess is staying
15		the same. So if I were to go even bigger than . . . if I were to go from the
16		twenty to the fifteen I still all I did . . . the twenty and the fifteen and
17		enlarge it even more its still going to be just that forty percent different
18		. . . difference.

Richard was having difficulty explaining his solution during this interaction. He initially focused on number relationships and did not speak about the quantities of base and height; nor did Richard discuss how these quantities were related. Richard also gave an incorrect description of the actual problem since he found what percent 6 is of 15 instead of determining what percent 15 is of 6 (Line 8) but described that the photo was enlarged by forty percent (Line 10). Richard did make correct observations (e.g., the total amount the values increase is different and the numbers increase by the same percent) (Lines

13–15), but note that he did not make the connection that since the photo is enlarged proportionally, the ratio of height to base would need to remain constant for any enlarged photo. Rather, he described, "that the overall way they are changing I guess is staying the same." Although Richard had difficulty in explaining his solution and also gave an incorrect explanation, it did appear that he believed he should give a justification of his solution. Thus, Richard was not violating the social norm that participants should *speak with meaning*. After Richard's comments, Jason invited other members of the PLC to ask questions to reveal Richard's thinking (*Excerpt 3*).

Excerpt 3.

19	Jason:	Mm k. Now you two (points the other two members) he's your student
20		and he just explained that to ya and remember we're just talk. . . we're
21		just trying to find out what he knows about constant ratio. We are given
22		two quantities . . . k? What are ya . . . how do you want to interview him?
23		What would you say? Either one.
24	Beth:	Could you come up with a statement that would show the relationship
25		between the height and the width that would work in any situation?
26	Richard:	Betwe . . . a relationship between the height. . . . (looks at Beth)
27	Beth:	(Interjecting) Between the length and width of this photo that would
28		work in any situation? That would you could always.
29	Richard:	Ya. Ya . . .and actually when I was . . . just thinking about that . . . ya
30		know the six is . . . if I were reducing it that the forty percent works what
31		I would need to do is take uh . . . six and divide that into fifteen to find
32		out what the . . . to find out what the real change in percentage was cause
33		I was enlarging and not shrinking it what I . . . what I said was if I was
34		shrinking if I were shrinking it going from fifteen to six then I just
35		shrunk the fifteen forty percent but I didn't necessarily enlarge the six
36		forty percent.

In the preceding excerpt, Beth listened to Richard's construction and made an effort to pose a question to encourage him to speak more meaningfully about his solution. This was apparent when Beth encouraged Richard to attempt to generalize the relationship between the base and height of the photo, thus requiring that he utilize the constant ratio between the base and height rather than finding a percentage change (Lines 24–25). After Beth's question, Richard continued to attempt to explain his reasoning. He began speaking about the relationship between the quantities and realized that his earlier comments were incorrect (Lines 32–36), although his continued comments were still vague and he continued to have difficulty providing a rationale for his statement that both the height and base will change by the same percent. The conversation of the PLC members continued in *Excerpt 4*.

Excerpt 4.

38	Richard:	So what I would do. . .I . . . might. . .what I would say is something like,
39		uhhh, setting it up like a function is what your asking?
40	Beth:	Like if you know one side is always bigger. . .
41	Richard:	(talking over Beth) Like just say the . . . like your f of x is, uh, fifteen
42		over six times x because the fifteen over is gonna be the same thing as
43		the twenty over eight . . . which . . . um . . . lets see that's two point
44		something.
45	Jason:	And then you did the fifteen over six because that was your base or
46		height or?
47	Richard:	Cause that's the enlargement. That's the, that's the bases . . .
48	Jason:	The bases.
49	Beth:	(Interjecting both Jason and Richard) What if I give a different
50		enlargement? What if I say I have a frame that's forty-two inches?
51	Richard:	Uh. So going from the six to forty-two? (looking at Beth)
52	Beth:	So now what will the height be (motion with hands upward) if I make
53		my base now forty . . . so I make the width forty-two? Can you come up
54		with number so that I can always . . . I have one side I can always figure
55		out what the other side is (motioning hand vertically and horizontally).
56	(long pause)	
57	Jason:	Doesn't matter what the number is . . . doesn't matter what the base is
58		that you start out with. Is there a function?
59	Beth:	A number that will always work. Is there some relationship there that
60		will always work . . . preserve this relationship between the width and the
61		height
62	Richard:	Well the eight is . . . you just put . . . I what I would say is just eight over
63		six . . . which is four thirds. So four thirds . . .
64	Beth:	So if I know . . .
65	Richard:	So that so that . . . when I get bigger . . . the forty-two inches that is going
66		to be on the bottom. What ever is on the top . . .
67	Beth:	Will be?
68	Richard:	Can't be any bigger than the four thirds when reduced . . . whatever
69		that decimal answer is.
70	Beth:	So if this is now if this is now forty forty-two (drawing on her paper) are
71		you saying that this (referring to her drawing) would be four thirds of
72		forty two.

73	Richard:	No I'm saying that . . . that uh the ratio of the forty-two and that length
74		will be the same length will be the same as four thirds.
75	Beth:	So if I multiply forty-two times four thirds will I have this length
76		(referring to her picture)? Cause I multiply the six times four thirds
77		would I get eight?
78	Richard:	Yeah
79	Beth:	So now we have the ratio . . . the relationship between the two sides
80		that we could use in any situation.

During the preceding interaction, Beth contributed to the sociomathematical norm of *speaking with meaning* by consistently referring to the quantities as base and height and by probing Richard to justify his explanations and generalize his solution. Although Richard gave a correct answer to the initial problem, Beth revealed, by persisting in her probing, that when speaking about the relationship between base and height of the photo she believed it was necessary for the group to describe the function relationship between the two quantities. Although Richard struggled to make this connection (Lines 41–44 and 68–69), she continued to question him until she felt the group determined the ratio and function relationship between the two quantities. With the help of Beth's questioning, Richard was able to find the ratio of four thirds that would work in any situation (Lines 73–74). With this ratio revealed, Beth again returned to the context of the problem in order to discuss how the ratio could relate one quantity to the other (Lines 75–77). Beth continued by describing how they had established a general relationship between the covarying quantities in the photo enlargement problem (Lines 79–80).

In summary, Beth appeared to listen to Richard's constructions and her persistence in prompting conveyed an expectation that continued probing was needed until it appeared that Beth believed Richard could *speak with meaning* about the quantities and how they were related. Richard began focusing on the ratio Beth intended (Line 62), but Beth continued to push him until the group had a solution that used the ratio in the context of the problem (Lines 75–77 and 79–80), although it was only Beth who provided this solution. Furthermore, after this ratio was determined, the group then made the move to write the function relationship between height and width on the white board they had available.

The PLC members appeared to be negotiating what constituted *speaking with meaning* for their group. The group was expected to use the quantities involved in the problem, which in this case were the base and height of the photo, and Beth persisted in questioning Richard until he was able to give a mathematical explanation for his solution. The discussions and probing continued until the members of the group were able to identify the constant ratio or constant multiplier that described how the quantities of base and height were related, and they were able to use this to define a functional relationship between the two quantities. The PLC members appeared to be establishing criteria for what would be considered *speaking with meaning*.

We argue that *speaking with meaning* becomes normative for a PLC when it emerges as criteria for closing a conversation. In other words, the PLC members continue to probe until the idea, concept, or problem has been discussed and/or justified by *speaking with meaning*. These criteria for closing a conversation are illustrated in the following example.

This example from the PLC occurred late in the semester. The group was discussing distance versus time graphs created by walking towards and away from a motion detector at varying rates. The facilitator, Jason, drew a graph on the board (Fig. 1) and asked the group to determine what kind of walking strategy would be necessary to create a similar graph (*Excerpt 5*).

Excerpt 5.

1	Jason:	What's the covariation of distance and time? What would you tell me
2		about increasing – using the terms increase or decrease and rate?
3		Carroll?
4	Carroll:	Well it looks to me like the person who's walking this is walking slowly
5		at first and then they increase their speed very quickly over a short
6		period of time and it gets faster and faster as they go.
7	Jason:	Okay. Would you agree? Andrew?
8	Andrew:	Yes. . .I agree. Um, the amount of distance they're going to cover in an
9		amount of time is continually increasing, so therefore their speed is
10		going to increase.
11	Jason:	Okay. How does this differ to constant rate?
12	Richard:	Well the constant was just pretty much going every. . .they were never
13		speeding up or slowing down. The rate was never increasing or
14		decreasing, it was always staying the same. So we were just, if we were
15		to graph it, we would just graph a straight line. It would just be the same
16		between this amount you went say this far (Motioning horizontal
17		distance versus vertical distance with his hands). This much time we
18		went this far up, this amount of time we went this far up (indicating
19		equal amounts of distance with hands). That's never changing, how far
20		we went or how fast we were going.
21	Beth:	(Stands up and goes to board.) This one you have this big amount of
22		time at the beginning but you only go like this little amount of [distance]
23		and then over here you have this little amount of time and then you go
24		this huge amount (demonstrating with graph on board by marking time
25		amounts along the horizontal axis and marking the corresponding
26		amounts of distance on the vertical axis which were sequentially getting

27	larger, Fig. 2). So the relationship for a certain amount of time is
28	changing all the time.
29	(The group moves to another topic.)

As revealed in this interaction, the PLC members continued their discussion of the graph until they justified the shape of the graph by describing how the quantities, distance and time, changed in tandem. While Carroll's initial solution was correct (Lines 4–6), she did not include an explanation of how the quantities covaried. The facilitator, Jason, then turned the discussion to the group by asking Andrew if he agreed with Carroll (Line 7). Andrew agreed and without further prompting responded by giving an explanation that attended to the quantities of the situation and how they were related. He specifically referred to the relationship between the distance traveled and the time traveled in the given graph (Lines 8–9). Then, after Jason inquired to how this graph differs from a constant rate, Richard attempted to describe a constant rate by discussing the graph of a straight line (Lines 12–20). However, Richard did not speak of the quantities involved and Beth spontaneously *spoke with meaning* by using the graph to show how the two quantities covaried and how for an amount of time, the distance traveled differed (Lines 21–28). These unsolicited comments demonstrated how *speaking with meaning* was being enacted as a sociomathematical norm in this PLC discussion.

This interaction (*Excerpt 5*) also illustrates how the facilitator began to direct conversations to ensure that the final explanation accepted by the group contained the quantities involved in the given problem. In contrast with the first example (*Excerpt 1*), where Jason's expectations for a sufficient explanation were vague, he tells the group that their explanation needs to address increasing and decreasing rate in terms of the quantities that are covarying (Lines 1–2). Making these expectations explicit contributed to the sociomathematical norm of *speaking with meaning*. Jason made a further move to have the group meaningfully describe the graph by asking them to contrast the presented graph with what would have happened if someone had been walking at a constant rate. Richard was able to identify how the quantities of distance and time would covary in this situation (Lines 15–19). In this discussion, the group was negotiating counterexamples as further criteria to be observed in order to *speak with meaning*.

In summary, the above results show that the social norms that focus on participation within the PLC are important for creating an environment where the teachers are supported in making sense of each other's solutions and strategies. Our results also support that within the PLC environment of this project, teachers are making progress in initiating and sustaining *speaking with meaning* as a sociomathematical norm. During initial sessions, such as that presented in *Excerpt 1*, the PLC
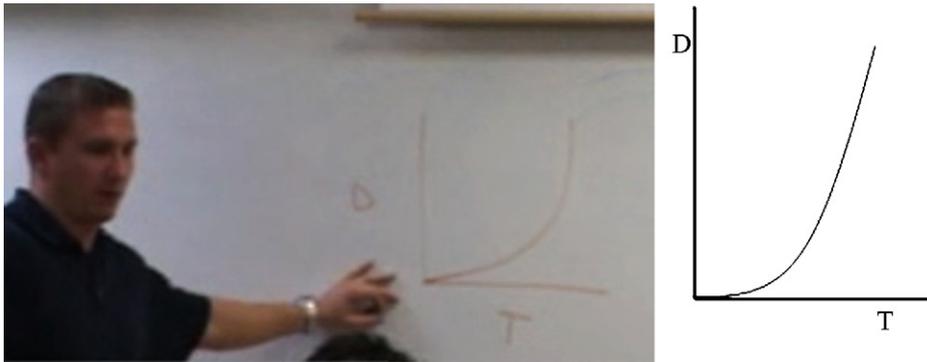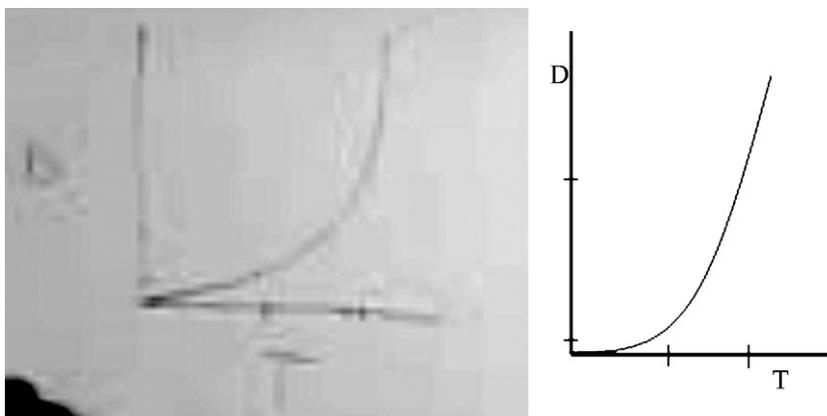

Fig. 1. Jason's distance and time graph.

Fig. 2. Amounts of time and distance.

members often gave procedural and vague explanations and the facilitator did not probe the members in order to promote *speaking with meaning*. Often, members would agree with vague explanations and then move on to a different topic. Thus, it appeared that the group did not have well-established criteria for what constituted *speaking with meaning*. As the semester progressed and the facilitator received weekly training, PLC members contributed to the social norm that participants should *speak with meaning* and it appeared that members were becoming comfortable in challenging each other when they observed other members acting outside of this normative behavior. For instance, members began probing each other in order to promote referencing quantities. Thus, the PLC members also appeared to be negotiating what constituted *speaking with meaning* for their group. This enabled them to develop criteria by which they could judge explanations as to whether or not they were sufficient. Eventually, the PLC group appeared to establish *speaking with meaning* as a sociomathematical norm where the group expected the members to participate in meaningful discourse before closing the discussion of a topic. For instance, it became expected that members provided explanations and justifications that were grounded in the context of the problem (*Excerpt 5*).

Also of importance, the weekly training sessions and workshops appeared to enable Jason, the facilitator, to gain a clearer image of the discourse referred to by *speaking with meaning*. During the PLC meetings, the responsibility of encouraging the teachers to *speak with meaning* within their PLC discussions fell primarily on the facilitator, Jason. Early in the semester Jason made few attempts to promote *speaking with meaning*. He was frequently observed allowing explanations that were fragmented and incoherent go unquestioned. As the semester progressed he was more frequently observed supporting the emergence of the sociomathematical norm by leading discussions about *speaking with meaning*, modeling *speaking with meaning* while presenting his solutions, and by probing PLC members about their solutions. Jason also began to encourage other members of the PLC to probe each other's reasoning and explanations, an action that did not occur early in the semester. It was not until after interventions with Jason that he began to facilitate the norm of *speaking with meaning* through these methods. These interventions consisted of the 3-h meetings throughout the semester and the weekly coaching sessions described in Section 4.

6. Discussion

It was important for us to be aware of some of the obstacles that may emerge when attempting to establish PLCs that engage in meaningful discourse concerning mathematics teaching and learning. Based on our past study of PLC meetings, we observed that participants were not engaging in meaningful discourse. PLC members' answers were frequently left unjustified and solutions were not given within the context of the original problem. Based on these observations, it became evident that the instructor of the graduate course needed to model the behaviors we hoped would become normative with the teachers and that these would guide their interactions during class and PLC meetings.

As an exercise in the first class of the semester, the instructor, in conjunction with the class, negotiated how communication should look within a PLC and the course. The teachers expressed that they valued meaningful communication and decided that it would be desirable if all members of the class attempted to speak conceptually when discussing their ideas during class. From their negotiation emerged the term *speaking with meaning*. While the class was in agreement that it is important to speak meaningfully about mathematics, their conception of what it meant to *speak with meaning* about mathematics was insufficient as revealed by our initial observations of the PLC meetings. As a result, we found it important that the facilitator encouraged members of the PLC to probe each other when they believed the conversations or explanations were not meaningful. In order to promote high standards for PLC communication, we found that the facilitator was required to be proficient in listening and questioning other members of the PLC. Analysis of our data revealed that the PLC facilitators did not initially possess these listening and questioning abilities. This resulted in our holding extra PLC training sessions during the semester, with the intention of better enabling the facilitators' promoting of *speaking with meaning*.

Through the coaching of facilitators on how they could promote *speaking with meaning* in their own PLCs, the notion of *speaking with meaning* created a focal point for training the facilitators. This resulted in *speaking with meaning* (as a phrase) emerging as a tool the facilitators could use to encourage PLC members to speak conceptually about the mathematics under discussion. Often the facilitators reminded the group that they should "*speak with meaning*" when they offered their solutions or suggestions. This tool resulted in helping the facilitators manage the discourse of their PLC meetings.

Speaking with meaning as a sociomathematical norm emerged from interactions among the members of a PLC. Thus, although the facilitator was trained to better understand the expectations for the discourse during a PLC meeting, this training was not sufficient in promoting PLC members to consistently *speak with meaning* as we intended. During initial PLC sessions, PLC members attempted to explain their own procedures but these explanations remained computational in nature, often incoherent, and each member remained focused on her or his own ways of thinking. It was not until interventions with the facilitator that we began to observe the members establish criteria for what would be sufficient for *speaking with meaning*. We found that each individual group discussed and negotiated what would constitute *speaking with meaning* for their PLC and it was this emergent activity that we described as the sociomathematical norm of *speaking with meaning*. It is noteworthy that what a PLC group constitutes as *speaking with meaning* will vary among different PLCs.

We also note that this work is not intended to claim that *speaking with meaning* was established in the PLC, but rather it was continually being negotiated and renegotiated. It was the criteria for what constituted *speaking with meaning* that was being established, subsequently bringing greater clarity to its meaning for the members of the PLC. As these criteria became established within the PLC members through this interactive constitution, each member of the PLC was able to deepen her

or his understanding of what it means to *speak with meaning*. Thus, we do not claim that our above criteria of *speaking with meaning* are exhaustive.

As previously noted, *speaking with meaning* had become a tool that facilitators and PLC members could use to encourage substantive mathematical discourse. We later found that *speaking with meaning* also emerged as a tool that teachers could use in their own classrooms to encourage their students to speak conceptually about topics of discussion. In discussions with teachers and administrators, we were told of several instances in which the phrase *speaking with meaning* was used in classrooms and appeared to have a positive influence on the mathematical discourse in the class. In fact, during an interview with one administrator, she spontaneously gave a detailed account of a distinct shift she had observed in the classroom discourse of one of her teachers. She recalled how she had visited this teacher's class prior to his participation in this project and characterized his teaching as "mind numbing" with him speaking and the students sitting quietly while they took notes. In contrast, she described having visited his class toward the end of the second semester of participation in the project. She indicated that he had the phrase "*Speak with meaning*" posted on his board. She also reported that the students were regularly challenging each other to *speak with meaning*, and that on several occasions the students had challenged the instructor to *speak with meaning*. She described this shift as a "big improvement" over her previous observation during which there was little student involvement and very few conceptual conversations about the content.

We note that while *speaking with meaning* was significantly influenced by the notion of conceptual orientation (Thompson et al., 1994), there is a distinct difference between the two constructs. While conceptual orientation refers to an individual's beliefs, attitudes and images of mathematics, *speaking with meaning* refers to the interactions that occur between individuals. Now, while we believe that an individual who has a conceptual orientation will be more likely to *speak with meaning*, we do not claim that every individual who has a conceptual orientation spontaneously *speaks with meaning* when communicating with another. That is, it is *possible* for an individual with a calculational orientation to *speak with meaning*. However, we believe that individuals for whom *speaking with meaning* becomes *normative* are likely acquiring a conceptual orientation in their mathematics communication patterns.

We also conjecture that there is a link between an individual's ability to *speak with meaning* and their content knowledge (relative to the subject they are speaking about). For example, it appeared that Beth and Andrew, the two mathematically strongest members of the group, were better able to *speak with meaning*. They were frequently observed providing explanations that referenced the changing nature of two covarying quantities and we conjecture that this was possible because of the understandings of proportionality and rate of change that Beth and Andrew both held. Our insight to their conceptual understanding comes from their high scores on homework and exams and their responses and contributions during the course. We highlight this possible link because we believe that *speaking with meaning*, and subsequently promoting *speaking with meaning*, is greatly enhanced by deep conceptual knowledge. This stance is consistent with Cobb, Wood, Yackel, and McNeal (1992) description of explanation as "an attempt to communicate aspects of their mathematical thinking that they think are not readily apparent to others." This notion of explanation implies that an individual's explanations can only clarify those things that the individual understands. Thus, deeper conceptual knowledge can enable and promote more meaningful discourse. While this is not the focus of this paper, it is an important aspect of *speaking with meaning*.

7. Implications

By creating the construct of *speaking with meaning*, which emerged from the activity (or lack thereof) of past PLCs, the research team emerged with a tool to inform interventions with PLC facilitators, with the intention of influencing the way the PLC members communicated with one another. The PLC members' activities that emerged from this intervention will help us illustrate exchanges that emerge within PLCs. Our observation of activities that emerged as normative allows us to connect our research with past research on mathematics in social settings.

Speaking with meaning is similar to a sufficient mathematical justification or explanation (Yackel & Cobb, 1996) in that it determines what will count as an acceptable form of communication. Just as what is a sufficient mathematical justification can be negotiated in a classroom, what counts as *speaking with meaning* can be negotiated in a PLC. However, we use *speaking with meaning* to refer to an intended outcome of *all* communication of an individual within a class or PLC setting, not just justifications or mathematical arguments. For instance, PLC members were also expected to *speak with meaning* about mathematical ideas, mathematics teaching and student learning. Also, *speaking with meaning* was used to help train facilitators so they could better manage the discourse of their PLC meetings. The term also carries with it the ability to operationalize what is a sufficient explanation by making clear to the audience what constitutes a sufficient explanation, thus becoming a tool that the teachers could use to challenge one another and take with them into their own classrooms (e.g., requiring that students "*speak with meaning*"). Thus, *speaking with meaning* has the dual nature of being both a theoretical construct and an intervention.

Further, *speaking with meaning* can provide both teachers and researchers a lens with which they can view the effectiveness of their efforts at promoting meaningful discourse. Researchers will be able to use this construct to gauge the level of mathematical discourse in the teachers' own classrooms. Teachers are expected to carry *speaking with meaning* and some of the behaviors to promote meaningful discourse into their own classrooms. By observing teachers and their students using the construct *speaking with meaning*, we conjecture that they are creating an atmosphere of sense making with regards to each other's thinking, as promoting *speaking with meaning* requires one to make another's discourse an object of reflection in order to gauge its meaning.

Our study of the emergence of *speaking with meaning* in a PLC setting has made us more aware of the actions within a PLC that contribute to quality discourse among the PLC members. This knowledge will inform our training of facilitators so that they are better prepared to promote *speaking with meaning* by members of a PLC. Since the facilitators are peers of the their PLC members, they typically do not have the extensive formal training to adequately promote these normative behaviors. Therefore, appropriate training methods and interventions are very important in preparing the facilitators to manage the discourse in their PLC meetings. Our awareness of the actions that produce *speaking with meaning* and quality discourse among members of the PLC will contribute to our refinement of the training methods and intervention materials.

In order to further investigate the influence of *speaking with meaning*, we will continue to follow the current PLC groups as well as study other newly formed PLC groups to continue investigating the effectiveness of our interventions. Other related research will include following the teachers into their own classrooms to investigate the degree to which behaviors of the teachers within the classroom mirror their activity in the PLC groups. We anticipate that these investigations will reveal new knowledge of the effectiveness of our PLC design in affecting teaching practice, while also informing the design of future PLCs.

Appendix A

A.1. The box problem

Given a rectangular piece of material, you can construct an open box by cutting equal squares from the corners and turning up the sides. Using an 8.5 in. by 11 in. sheet of paper, construct a box.
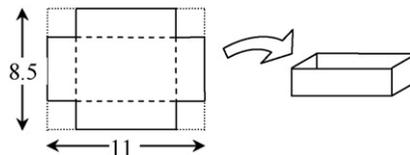
A.2. Photo enlargement problem

A photograph that is 6 in. on the base and 8 in. high is to be enlarged so that the new base is 15 in. What will the height of the enlargement be?
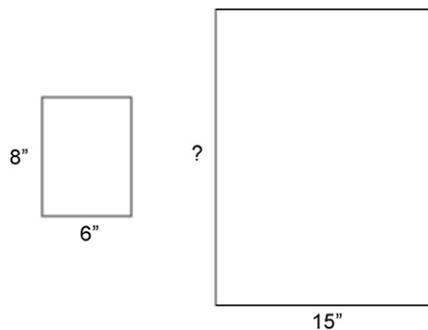
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