Design Considerations in Complementing Graphical Reasoning Research with Eye-Tracking Technologies

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Moore, K. C., Yasuda, S., Henry, M., Çaylı, S., & Mavengere, T. (2025). Design considerations in complementing graphical reasoning research with eye-tracking technologies. In R. M. Zbiek, X. Yao, A. McCloskey, & F. Arbaugh (Eds.), *Proceedings of the 47th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 1595-1604). The Pennsylvania State University.

Available at:
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DESIGN CONSIDERATIONS IN COMPLEMENTING GRAPHICAL REASONING RESEARCH WITH EYE-TRACKING TECHNOLOGIES

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Building on our prior work investigating individuals' quantitative and covariational reasoning (QCR), we explore complementing methods we have previously used with those involving eye-tracking technology. Eye-tracking technology provides insights into attentional focus, and these insights remain unclear in their application to and association with QCR research. Exploring these applications and associations presents methodological challenges and considerations. We discuss these challenges and considerations with a focus on defining areas of interest (AOIs), modifying graphical tasks, and structuring interview protocols. By making these challenges and considerations explicit, we contribute to the broader conversation on leveraging emerging technologies to complement extant methods that target mathematical reasoning.

Keywords: Research Methods, Mathematical Representations, Cognition, Technology

Research on individuals' *quantitative and covariational reasoning* (*QCR*) has proliferated over recent years due to its importance for mathematical reasoning inside and outside of math classrooms (Carlson et al., 2002; Karagöz Akar et al., 2022; Smith III & Thompson, 2007; Thompson, 2013; Yoon et al., 2021). Accordingly, this research base includes several stable constructs for explaining individuals' meanings in contexts related to QCR (e.g., Ellis et al., 2020; Johnson, 2015; Jones, 2022; Lee et al., 2020; Paoletti et al., 2023). Another emergent body of research involves researchers capitalizing on new technologies to investigate individuals' eye-behavior as they consider and solve mathematical tasks. This includes exploring eye-mind relationships (e.g., Schindler & Lilienthal, 2019; Thomaneck et al., 2022). Although most researchers accept that eye-behavior does not absolutely indicate thinking, researchers have demonstrated that an individual's eye-behavior can provide viable insights into some aspects of thinking (Strohmaier et al., 2020). This feature, in tandem with the growing importance of individuals' QCR, has led to researchers including ourselves considering the ways individuals' eye-behavior relates to their QCR (Thomaneck et al., 2022; Waters et al., 2019).

Any research base that takes on new methodologies is presented with challenges and considerations. These challenges and considerations often go unreported, ultimately inhibiting the dissemination of research (Drimalla et al., 2020; Tyburski et al., 2021). In this report, we give attention to the challenges and considerations we have had to make as we have worked to incorporate an eye-tracking focus in relation to individuals' QCR in graphical representations. We discuss our approaches to designing and defining *areas of interest* (*AOIs*) and to modifying interview protocols to target collecting eye-tracking data. Although the challenges and considerations may not be novel amongst the entire field of eye-tracking research, we contribute novel insights by situating such phenomena in the context of investigating individuals' QCR in graphical representations.

Zbiek, R. M., Yao, X., McCloskey, A., & Arbaugh, F. (Eds.). (2025). *Proceedings of the forty-seventh annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Pennsylvania State University. https://doi.org/ 10.51272/pmena.47.2025

Background and Context

The present work draws on our prior two decades of work investigating individuals' QCR inside and outside the classroom (Moore, 2021; Moore et al., 2022; Moore et al., 2013; Moore, Stevens, et al., 2019). Much of this work has occurred in the context of graphical representations. As part of this work, we have drawn heavily on Moore and Thompson's (Moore, 2021; Moore & Thompson, 2015) constructs of *emergent graphical shape thinking* (*EGST*) and *static graphical shape thinking* (*SGST*). EGST involves an individual reasoning about a curve/line as a trace produced by uniting two quantities and creating a record of their covariation. SGST involves an individual reasoning about a curve/line as an object in-and-of-itself with perceptual and thematic properties of shape. Mathematical properties are derived from quantities' covariation with the former and learned facts and shape patterns with the latter (Moore, 2021; Moore & Thompson, 2015). For instance, an individual might understand a linear relationship in terms of how one quantity changes in a multiplicative relationship with the other, or they might associate linear relationships with lines that are grouped by general direction (e.g., horizontal, vertical, sloping upward/downward left-to-right).

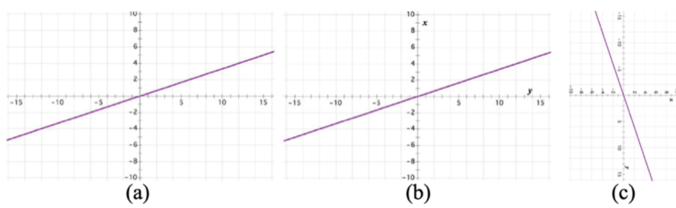
An individual's eye-behavior is necessarily tied to their cognitive activity. Although an individual's cognitive attention does not exclusively define where their visual attention lies, an individual can choose where to look and visual stimuli provide material to give attention to. It thus stands to reason that the meanings individuals draw on have some relationship with their eye-behavior (Carter & Luke, 2020). This is the hypothesis we are interested in pursuing. Inspired by Waters et al. (2019), who studied associations between students' QCR and their frequency of switching between AOIs, we seek to conduct a more fine-grained analysis of students' graphical shape thinking and their eye-behavior. Specifically, we investigate (a) In what ways can eye-tracking technology be used to complement current methodologies for investigating and supporting students' graphing meanings? and (b) In what ways is attentional focus related to students' graphing meanings?

Researchers have used eye-tracking technology to gain insights into stakeholders' activities in various educational contexts: student interpretation of mathematical representations (e.g., Boels et al., 2025; Bolden et al., 2015; Hahn & Klein, 2025), proof comprehension (e.g., Beitlich et al., 2014; Roy et al., 2017), pedagogical content knowledge (e.g., Brunner et al., 2024), and teacher noticing (e.g., Kosko et al., 2024), to name a few. Such studies primarily involve the analysis of fixations and saccades, which requires that researchers define AOIs for prominent objects in a visual stimulus. A fixation is that which falls inside of an AOI, and saccades can then be defined as movements from one AOI to another. When using static visual stimuli, most researchers have chosen relatively simple AOIs to make the categorization of fixations straightforward. For instance, researchers have used AOIs to encompass entire representations such as a graphical display, equation, or written description (e.g., Beitlich et al., 2014; Bolden et al., 2015). Our focus on individuals' constructed meanings within a graphical display inhibits our taking this approach. Whereas global approaches treat a graphical display as a single object (e.g., Hahn & Klein, 2025), we require a more complex discernment of visual stimuli including but not limited to axes, labels, and a curve. We thus face the challenge of how to define appropriate AOIs and how to modify available QCR tasks and protocols for our eye-tracking purposes.

Considerations in Combining Eye-Tracking and Graphical Shape Thinking

We organize our considerations around two themes: considerations when defining AOIs and when designing interview protocols. We draw on a task (Figure 1) presented by Moore, Silverman, et al. (2019) to discuss our considerations. Figure 1a is presented to the participant as a potential solution to $y = 3x$. They are given a prompt like: “You are working with a student who happens to be graphing $y = 3x$. He provides the following graph. How might he be thinking about the situation?” As the participant responds, the interviewer asks questions consistent with a semi-structured clinical interview protocol for the purpose of eliciting their thinking. When the interviewer senses the participant has exhausted the potential hypothetical ways of thinking for the solution, the interviewer provides Figure 1b and a prompt like: “What if, while working with the student, he did the following. What do you think of this solution and how would you respond to the student?” The interviewer again asks questions for the purpose of eliciting the participants’ thinking. The participant often rotates the given graphical display (which is provided on paper) 90º clockwise or counterclockwise to obtain a horizontally-oriented x-axis (Figure 1c). If the participant does not do this, the interviewer eventually does so and asks the participant to consider that orientation as a solution to $y = 3x$. Throughout the multi-part task, the interviewer is given the freedom to ask spontaneous follow-up and probing questions based on the participant’s actions. The goal is to develop a model for the participant’s in-the-moment meanings, and the interviewer asks questions based on their expertise and knowledge to accomplish this goal.

Figure 1: A Student’s Solution to Graphing $y = 3x$ (Moore, Silverman, et al., 2019, p. 185)



The task aids a researcher in distinguishing between meanings that foreground covarying quantities and those that foreground figurative aspects of slope such as horizontal runs and vertical rises (Moore, Silverman, et al., 2019; Moore et al., 2024). The former meaning enables conceiving Figure 1b as conveying a relationship such that y changes by 3 times the amount x changes, while the latter can result in a participant rejecting the solution due to $y = 3x$ requiring a vertical rise of 3 for a horizontal run of 1. In terms of the 90º rotation (Figure 1c), a researcher is aided in distinguishing between the aforementioned meanings. The former meaning enables conceiving Figure 1c as still conveying a relationship such that y changes by 3 times the amount x changes, while the latter can result in a participant rejecting the “negative slope” line due to $y = 3x$ requiring a line that rises from left-to-right (i.e., a “positive slope”).

AOI Considerations

In modifying the tasks for eye-tracking purposes, we encounter several interrelated concerns regarding the design and placement of AOIs. These involve considering (a) the number of stimuli, (b) the proximity, (c) the size of stimuli, (d) potential relationships between saccades and stimuli, and (e) determining formal AOIs.

Number of stimuli. One of our first considerations relates to the number of stimuli provided with each task. Having an abundance of stimuli can limit the power of one’s analysis, particularly if much of the stimuli is insignificant in its relevance to the research questions at hand. Stimuli a researcher deems insignificant might attract participants’ attention, making eye-tracking data unnecessarily messy. Furthermore, an abundance of stimuli can inhibit the development of well-defined AOIs due to potential overlap in AOIs. We are also concerned with the extent to which particular figurative material provides stimuli that guide participant’s

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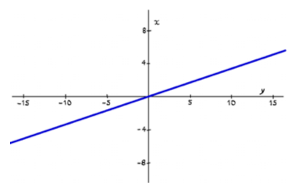
attention in direct ways or distracts the participants with mathematical concepts or meanings not necessarily related to the researcher's task intention.

Reviewing Figure 1b, we decide to remove the gridlines. Inclusion of the gridlines results in numerous areas of stimuli overlap and the gridlines provide figurative material directing the participant to-and-from the axes and curve. For this task, we also decide to reduce the number of available stimuli by reducing the number of marked and displayed values on the axes. Doing so reduces the number of potential AOIs and enables better delineating defined AOIs. We provide a graphical display that modifies Figure 1b with these considerations in mind (Figure 2).

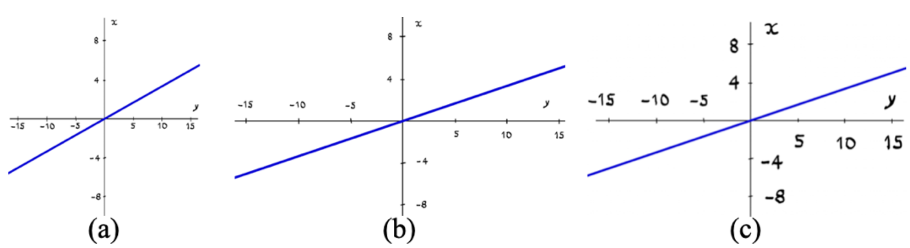
Figure 2: Modifying the Graphical Display in Figure 1b to Account for Stimuli Amount



Proximity. Intertwined with considerations regarding the number of stimuli are considerations regarding the distance between various stimuli. Having stimuli too close in proximity can inhibit the development of well-defined AOIs due to potential overlap in AOIs; having stimuli in close proximity inhibits defining the object to which a participant is giving their attention. We are also concerned that an abundance of stimuli in one area might guide participant's attention in direct ways. Similarly, we are concerned that having similar stimuli receive disproportionate proximity might also influence the participant's attention in unintended ways.

Returning to Figure 2, we consider the scaling of the axes. For example, we could decrease the proximity of the curve with respect to the horizontal axes as shown in Figure 3a, but in doing so we also increase the proximity of axes values. Scaling the axes also influences the visual properties of the curve, and in this case we desire to have the curve maintain its original look. We also consider the placement of the stimuli relative to other stimuli. Figure 3b illustrates decreasing the proximity of available stimuli with respect to the values in relationship to the axes and curve (but not with respect to each other) for the same axes scaling as in Figure 2.

Figure 3: Modifying Figure 2 to Account for Stimuli (a-b) Proximity and (c) Size

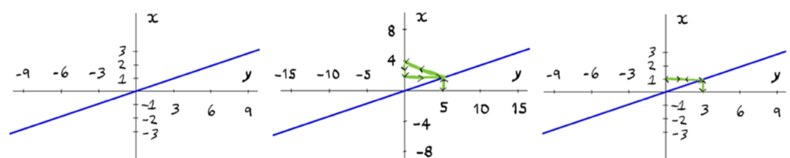


Size of stimuli. The interrelated nature of the considerations we are identifying is clear at this point. For example, as the number of stimuli is increased, it is likely that the proximity of stimuli is increased. An additional consideration that influences proximity and number of stimuli considerations is stimuli size. As the size of a stimulus increases, its proximity to other stimuli increases. Increasing stimuli size decreases available space for stimuli. Varying the relative size of stimuli also influences how salient they are with respect to each other, including the relative eye strain necessary to focus on the stimuli.

Starting with Figure 3b, the size of stimuli can be varied in numerous ways. With respect to the axes and curve, the thickness of each can be varied. With respect to the numerical and variable labels, font size can be increased or decreased. Depending on researcher intentions, the line widths and font size need not be consistent among stimuli. Figure 3c illustrates increasing the font size of the stimuli for the purpose of improving readability and decreasing eye strain, while keeping axes and curve thickness equivalent to that in Figure 3b.

Saccades and stimuli. Eye-tracking technology enables investigating AOIs and movement between AOIs. We are interested in participants' saccades due to the differences between EGST and SGST. Our prior research suggests an individual enacting EGST gives attention to how a curve's emergence is necessarily tied to how magnitudes covary in their Cartesian arrangement; they balance attention between the curve and its axes projections. With respect to eye activity, these attentional patterns might correspond to saccades from the curve to the axes and vice versa. SGST involves giving attention to the physical shape of the curve including properties related to direction and general curvature. Our prior research indicates that SGST does not entail attention to axes properties, and thus these attentional patterns might correspond to saccades occurring primarily along the curve itself. We intend to design stimuli that enable us to observe and distinguish between these hypothetical attentional patterns.

Figure 4: (a) Modifying Figure 3c to Take into Account (b-c) Hypothesized Saccades

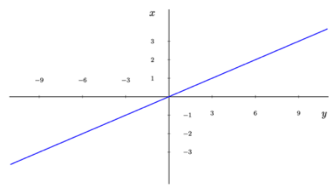


(a) (b) (c)

Returning to Figure 1b and after its modifications to produce Figure 3c, we consider two decisions. First, because we hypothesize that EGST involves attentional focus moving from the curve to the quantities' magnitudes or values on the axes, we first decide to provide labeled axes values that correspond to each other (Figure 4a). We also modify the values to minimize any needed calculations to determine the relationship's rate of change. We acknowledge that this design choice provides figurative material that might direct participants' attention from the curve to the axes. In previous work and pilot work, however, we did not find this to be the case. In fact, we observed that unpaired values frequently resulted in participants attempting to estimate paired values using the available values (Figure 4b). This created saccades that were consistent with our hypotheses yet added additional paths due to the unpaired nature of values (Figure 4b vs. Figure 4c). Relative to our purposes and the underlying participant reasoning, these additional paths act as noise that contributes to random variability in the data. Second, we desire the curve to be such that we can distinguish saccades that move from the curve to the axes from those that move along the curve; the display of each curve needs to be such that its proximity affords an AOI distinct from the coordinate axes. As mentioned above, this requires balancing the scale of the axes, the size of the stimuli, the number of stimuli, and the defined *x*-*y* relationship. For example, as the rate of change of *y* with respect to *x* is increased within a fixed axes orientation and scale, the curve increases in proximity to the *y* axis. Ultimately, we decide to maintain a linear relationship with a rate of change of 3 or 1/3 at the scale displayed in Figure 4c.

Determining formal AOIs. Taking into the previous considerations, we create Figure 5. We also include *x* and *y* on the same axes side as the values so that a participant's attention does not have to cross an axis if checking the values and variable referent. With this figure in hand, it is still necessary to define formal AOIs (and, if necessary, make additional modifications based on those regions). There is no consensus regarding the definition and use of AOIs among researchers (Orquin et al., 2016). Rather, there are various approaches, with these ranging from being strictly mathematical to a balance of theoretically and mathematically defined.

Figure 5: The Result of Taking into Account AOI Considerations



One standard approach is to define an AOI based on a distance from a stimulus. Although intuitive, even this standard approach is more complicated than first thought may make it appear. Do we use absolute distance from a stimulus? If so, do we use taxi-cab geometry or direct distances? Do we instead use an amount of angular sweep from the viewer's perspective to define a radial AOI around a stimulus? Under what conditions does an angular sweep approach create circles versus not around a stimulus? When creating such regions, do we use the center or edge of a stimulus as the place to measure a distance or angular sweep from? How do we determine a "center" or "edge" of a stimulus? We denote a subset of region options in Figure 6. These are questions that our group and prior researchers have each wrestled with in defining AOIs. We consider them critical questions to explicitly consider in order to convey our methodology to other researchers, as these decisions directly impact produced data and subsequent analysis.

Figure 6: AOI options for a displayed value.

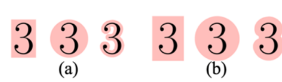
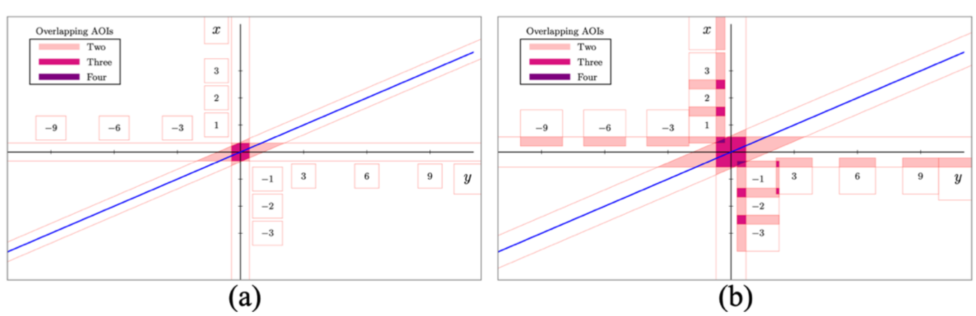


Figure 7: Sketching overlapping AOIs defined using directed distances.



In Figure 7, we illustrate the result of creating AOIs using fixed distances from stimuli edges to define rectangular enclosures. Both figures sketch the overlapping AOIs, with Figure 7b

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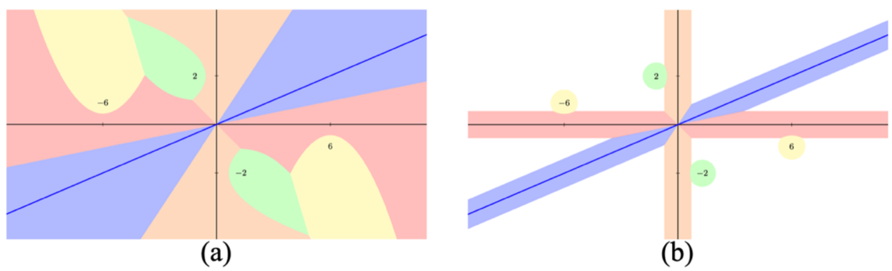
illustrating more overlapping regions due to using an increased directed distance. In Figure 8a, we illustrate the result of creating AOIs using the Voronoi tessellation method (Hessels et al., 2016) with the center of rectangular stimuli containments defining the point for each Voronoi region. For ease of illustration, stimuli include the horizontal axis, the vertical axis, the curve, and a subset of values. In Figure 8b, we illustrate the result of using a limited-radius Voronoi tessellation, which enables a researcher determined constraint. Figures 6-8 illustrate how one defines AOIs impacts the data collected and, hence, results and drawn inferences.

Figure 8: AOIs Using (a) Infinite and (b) Limited-Radius Voronoi Tessellation



Protocol and Methods Considerations

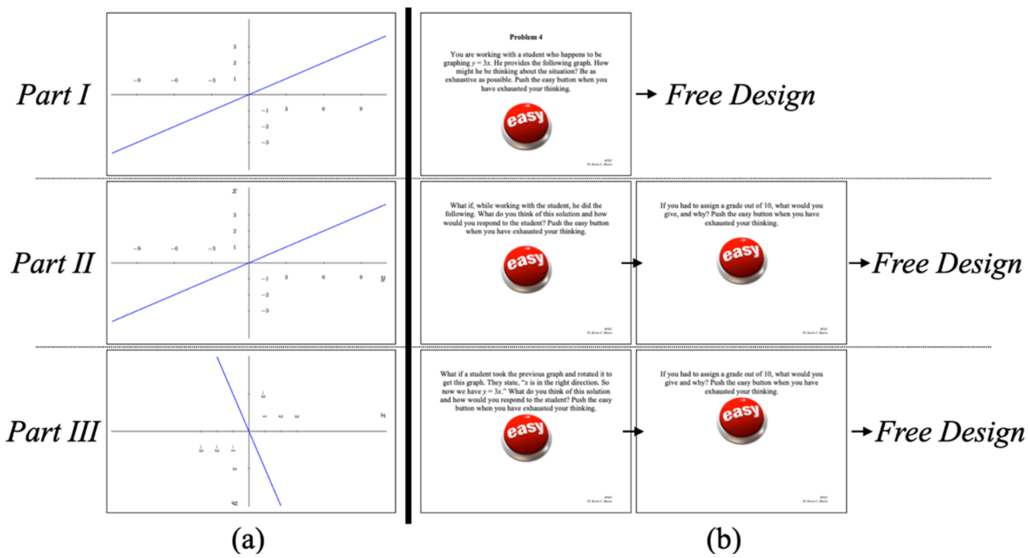
Our prior research on participants' graphing meanings has been qualitative with an extensive focus on building second-order models (Hackenberg et al., 2024; Ulrich et al., 2014) through a combination of semi-structured clinical interviews (Clement, 2000; Ginsburg, 1997; Goldin, 2000) and teaching experiments (Steffe & Thompson, 2000). These afford the researcher a certain amount of flexibility to ask follow-up, probing, and thinking-eliciting questions. At times these questions can be quite open, and at other times they can be quite narrow. A researcher's questions are informed by their expertise and intentions. A byproduct of this is that a participant's attentional focus can be drawn to stimuli by the researcher's questioning regardless of the participant's meaning and associated cognitive attention. Because a primary interest of ours is investigating participants' eye-behavior as it relates to their enacted meanings, we were concerned that the type of researcher questioning typical of our previous studies would impact our investigation in undeterminable and unquantifiable ways.

A solution to this is using a scripted interview. We hesitated to script the interview protocols in absolute. In our experience, developing viable second-order models of student thinking necessarily involves ad hoc questioning informed by the researcher's observations and expertise. We thus rely on a hybrid protocol design and for each task we design a scripted portion and a free-design portion. We return to the task built around Figure 5 to illustrate.

During the scripted portion, which occurs at the onset of a new task or new graphical display within a task (e.g., adding variable labels to axes as with moving from Figure 1a to Figure 1b), we dedicate the computer monitor to the graphical display (Figure 9a, Parts I-III) with the prompt provided on a sheet of paper given to the participant (Figure 9b, Parts I-III). Our choice to separate the graphical display and the prompts is so if focused on a prompt, the participant cannot be processing stimuli forming the graphical display using their peripheral gaze. Also, a printed prompt ensures that the participant can return to a prompt without having to request it from the researcher. We place an "easy button" (see Figure 9b) on the prompt page for the participant to press when they have exhausted their thinking. This action signals to the researcher to provide the next prompt or to begin the free-design phase of questioning, as indicated in Figure 9b. This design choice enables the participants to control the time spent considering a prompt and stimulus, thus allowing each individual sufficient processing time without the researcher having to engage them.

A free-design protocol follows the completion of each scripted portion. The researcher can ask probing, thought-eliciting questions based on their expertise and participant actions. We hypothesize that this flexibility enables us to develop viable insights into the participants' meanings. The scripted portions, on the other hand, provide us data to analyze participants' eye-behavior during their initial assimilation of a graphical display. Looking holistically, the scripted and free-design portions provide us two data corpuses: one data corpus in which the researcher does not intentionally influence participant focus and one data corpus in which the researcher does intentionally influence participant focus. This is not to say that participant-researcher interactions from one task do not carry over to another task. But a participant's first actions on a new task and graphical display are such that the participant must spontaneously make those associations, rather than the researcher possibly prompting them in that task context.

Figure 9: An Example Protocol For Scripted and Free Design Portions



Closing

We discussed various considerations and challenges we have encountered incorporating eye-tracking technologies into our research. We focused on AOI considerations and how we modify tasks with these in mind, as well as protocol and methods issues related to mitigating researcher influence on a participant's initial engagement with a stimulus. Our considerations are certainly not the only challenges and approaches available. This paper is a provisional solution to those challenges, merely acting as an aid in promoting the ongoing conversations related to incorporating newer technologies into education research. New technologies can transform our research in several ways, including addressing the resource intensive aspects of qualitative research; if associations between meanings and eye-behavior exist, then eye-tracking technologies and AI-driven analyses can develop viable characterizations of individuals' meanings at a rate currently impossible for human researchers. More generally, ever-evolving machine learning technologies will continue to open new ways to design studies and analyze data. These changes and evolutions will only occur in productive, insightful, and learner-sensitive ways if we take a critical and detailed lens to their development and use.

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