Relationships between Pre-Service Teachers' Decentering Actions and Quantitative Reasoning

Kevin C. Moore
Luke J. Zhuang
Cun Ma
Geon Lee

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Kevin C. Moore
University of Georgia

Luke Jiawei Zhuang
University of Georgia

Cun Ma
University of Georgia

Geon Lee
University of Georgia

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The emergence of quantitative reasoning—conceiving situations in terms of measurable attributes and relationships between those attributes (Carlson et al., 2002; Karagöz Akar et al., 2022; Thompson, 2011; Thompson & Carlson, 2017)—as critical for K-16 students' mathematical development has generated a need for exploring its role in preparing future teachers. Also related to supporting K-16 students' mathematical development is the act of decentering, which involves a teacher putting aside their own ways of thinking for the purpose of discerning and building a model of their students' thinking (Baş-Ader & Carlson, 2022; Carlson et al., 2022; Ellis, 2022; Hackenberg et al., 2024; Silverman & Thompson, 2008; Teuscher et al., 2016). Notably, researchers have recently illustrated an important link between teachers' decentering and quantitative reasoning capacities (Tallman & Frank, 2020; Tallman et al., 2024).

In this poster, we explore secondary mathematic pre-service teachers' (PSTs') decentering actions and their quantitative reasoning. Specifically, we draw on clinical interview data to illustrate relationships between the PSTs' enacted meanings and the ways in which they appraise secondary students' solutions and reasoning. The PSTs' enacted two primary types of meanings. As one type, they drew on meanings that consisted of declarative facts and indexical associations. For example, some PSTs drew on a learned fact that involved a type of function (e.g., a quadratic function) necessarily being associated with a shape (e.g., curving up). As a second type, they drew on meanings that involved enacting quantitative operations to make sense of the posed solutions and reasoning. These PSTs made sense of a graph by considering how it might have emerged via the covariation of the two quantities along the axes (e.g., y changes by three times the amount x changes). We noted that in some cases, PSTs drew on both types of meanings to make sense of the posed student solutions and reasoning.

With respect to the PSTs' decentering actions, their dispositions varied depending on the enacted meanings. When PSTs enacted meanings foregrounding declarative facts and indexical associations, they typically focused on the extent the posed solutions and reasoning either violated or maintained those learned facts and associations. For instance, if a given graph had a learned shape they associated with a function the secondary student intended to graph, they deemed it correct. If it did not have the learned shape they associated with the function, they deemed the student incorrect regardless of the student reasoning presented. When PSTs enacted meanings foregrounding quantitative reasoning, they predominantly searched for various ways posed student solutions and reasoning can be considered a viable and correct solution to the posed task. For instance, regardless of the familiarity of a given graph's shape, these PSTs considered conventional and unconventional ways in which the graph might convey the quantitative relationships the graph was to convey. This enabled them to account for unfamiliar student reasoning that was viable. In general, we illustrate that PSTs' quantitative reasoning provided a flexible grounding for considering students' diverse ways of reasoning as compared to instances in which PSTs enacted declarative facts or indexical associations.

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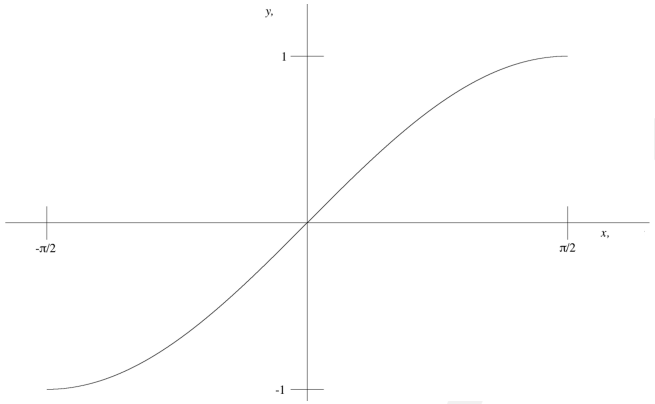
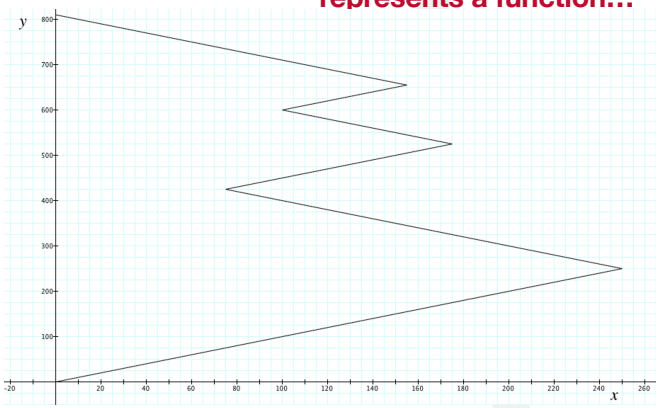
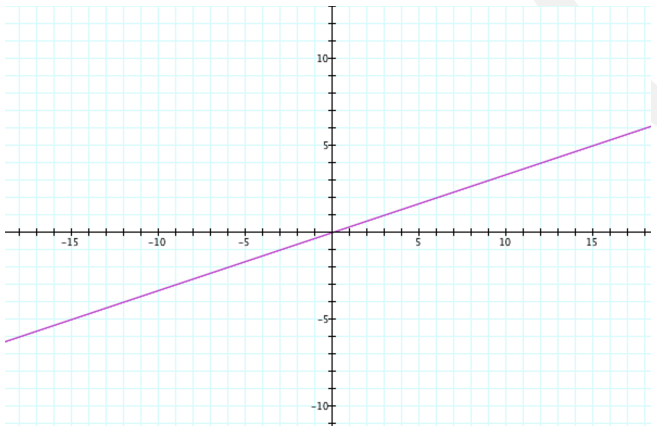
A student is determining if this could be inverse sine…



A student is determining if this graph represents a function…



A student is determining if this graph can be $y = 3x$…



In what ways does quantitative reasoning influence teachers' decentering capacity?

(1) What role does it play in teachers differentiating between representational practices and mathematical reasoning?

(2) How does quantitative reasoning impact their ability to honor students' initial construction of major mathematical ideas?

(3) How do we support teachers' quantitative reasoning so that they can both notice and build upon their students' quantitative reasoning and mathematical realities?

Problematized Extant Meaning	Enacting an Alternative Meaning	Comparing Extant and Alternative Meanings
Assimilation engenders a perturbation	Reconciliation of perturbation occurs via an accommodation	Reconciliation of perturbation occurs via an accommodation
Disequilibration occurs with respect to the task situation	Equilibration is established with respect to the task situation	Equilibration is established with respect to competing meanings
An intellectual need for an alternative meaning is created	An intellectual need for comparing extant and alternative meanings is created	An intellectual need for pursuing the implications of alternative meanings may occur

EXPLORE HERE

TQRCG

What student response do you prefer?

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Mary Frances Early College of Education
UNIVERSITY OF GEORGIA