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RESEARCH ARTICLE

Exploring the mathematical creativity of preservice teachers: A beyond conceptions approach

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Abstract

Mathematical creativity is a multifaceted psychological construct, necessitating innovative research techniques to enhance its integration into mathematics education. Though existing studies primarily focus on teachers' personal conceptions of mathematical creativity, there is a pressing need to identify their actual creative practices during problem-solving (PS) and problem-posing (PP) activities. Understanding these authentic acts of creativity is crucial for evaluating how teachers' thinking and enactment of mathematical concepts reflect their creativity and influence their instructional practices. In this paper, a theoretical and methodological approach to studying mathematical creativity in action by integrating two theoretical approaches to outline mental operations that underpin creative characteristics is proposed. We illustrate the application of our theory and methodology using examples from a specific study of pre-service teachers' (PSTs') PS activities. Our approach aims to fill the gap in understanding mathematical creativity as it unfolds, providing insights into cognitive processes and contextual factors affecting mathematical creativity, and ultimately informing the evolution of practices in mathematics education research and practice.

Keywords: mathematical creativity, theory, methodology, pre-service teachers.

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I. INTRODUCTION

Mathematical creativity is a multifaceted and complex phenomenon, thus necessitating innovative research techniques to understand and support its integration into the teaching and learning of mathematics. To date, research on teachers' mathematical creativity, whether in-service or pre-service, has mostly focused on their personal conceptions of mathematical creativity (e.g., Andrade & Pasia, 2020; Bolden et al., 2010; Leikin et al., 2013; Lev-Zamir & Leikin, 2011;2013; Shriki, 2010; Waswa & Moore, 2020). Understanding teachers' conceptions of mathematical creativity is important. Teachers' conceptions influence the decisions they make in designing and delivering instruction. These decisions, in turn, have a direct impact on their classroom environment, and, ultimately, on student creativity and learning outcomes.

Although existing research has primarily focused on teachers' conceptions of mathematical creativity, recognizing its pivotal role in shaping instructional decisions, a need to delve deeper into teachers' exhibited mathematical creativity as they engage in PS and PP (problem posing) activities exists. Going beyond teachers' personal conceptions of mathematical creativity to explore their authentic acts of creativity is crucial for several reasons. Firstly, it allows us to gauge the degree to which their thinking during PS/PP and the enactment of mathematical concepts manifests creativity. Secondly, it enables us to characterize how effectively their understandings facilitate their creative engagement with mathematics. Finally, it helps assess the alignment between their conceptualizations of mathematical creativity and the extent their PS/PP and understandings of mathematical concepts entail mathematical creativity. Ultimately, teachers' understandings and PS/PP activity can form a basis for their personal conceptions of mathematical creativity and, hence, influence what they strive to enact and encourage in their classrooms.

Researchers who have focused on understanding mathematical creativity of individuals (e.g., Bahar & Maker, 2011; Bokhove & Jones, 2018; Carreira & Amaral, 2018; Kwon et al.,2006; Lev-Zamir & Leikin, 2011, 2013; Tabach & Levenson, 2018; Vanegas & Giménez, 2018; Walia, 2012) have concentrated on determining two or more indicators of mathematical creativity. These indicators include fluency, flexibility, originality, elaboration, activation of mathematical knowledge, activation of representational means among others. These researchers have used various methodological approaches including analyzing products of individual solutions of nonroutine problems (e.g., Andrade & Pasia, 2020; Bahar & Maker, 2011; Carreira & Amaral, 2018; Kwon et al.,2006; Lev-Zamir & Leikin, 2011, 2013; Tabach & Levenson, 2018), investigating PSTs' noticing of students' creativity using sample cases of teacher-student dialogue (Vanegas & Giménez, 2018), and coding survey responses from mathematics professional (Moore-Russo & Demler, 2018).

A predominant theme in studies on understanding mathematical creativity is an extensive focus on analyzing products for mathematical creativity, such as analyzing students' solutions for a task that require creative-problem solving skills. A significant gap remains in understanding what we term *mathematical creativity in action*, which involves understanding creativity in the moment of a mathematical activity such as mathematical PS or PP. This approach to understanding mathematical creativity is crucial for several reasons, including but not limited to gaining insights into an individual's cognitive processes and strategies as their creativity unfolds, revealing contextual factors that affect an individual's mathematical creativity, and providing immediate feedback which can help an individual recognize and build their creative thinking processes as they occur. Aligning with our argument, Savic et al. (2017) emphasized that "researching the creative product may not provide full understanding of the development of creativity or may not reflect the creativity used to reach that product" (p. 25). One approach to addressing mathematical creativity in action is to focus on the cognitive processes underlying the observable characteristics of creativity. To achieve this focus, we propose a theoretical and methodological approach to understand mathematical creativity in action. Specifically, we integrate several theoretical constructs to operationalize mathematical creativity via outlining potential mental operations that underpin certain mathematical creativity characteristics/components. Building on this operationalization, we discuss a methodological approach that provides a viable way of investigating mathematical creativity in action.

We begin with a literature review of general creativity and mathematical creativity definitions. Next, we explore various perspectives on mathematical creativity and propose a theoretical approach to situating mathematical creativity in terms of particular mental operations. We then outline a methodological approach based on the outlined approach to investigate mathematical creativity in action. To illustrate our theory and methodology, we present examples from PSTs' PS and PP during open-ended tasks. Finally, we provide a brief discussion related to the implications of our proposed approach.

II. RELATED LITERATURE

Definitions of General Creativity

Before highlighting various definitions of *mathematical creativity*, we first summarize definitions of general *creativity*, as several researchers have adopted characteristics of general creativity in their study of mathematical creativity. Researchers (e.g., Haylock, 1987; Leikin, 2009; Liljedahl & Sriraman, 2006; Mann, 2006; Sriraman, 2005) have argued that there is no universal definition of creativity. Example of definitions of general creativity include those by Guilford (1967) and Runco (1993), Plucker and Beghetto (2004), Sternberg and Lubart (1999), and Silver's (1997) summary on two perspectives of mathematical creativity.

Guilford (1967) categorized creative abilities into divergent production and transformational abilities. Divergent production involves generating diverse ideas with by fluency, flexibility, and elaboration in PS. Transformation abilities involve revising existing knowledge to create new forms or patterns. Runco (1993), expanded this definition to include convergent thinking, problem finding and PS, self-expression, intrinsic motivation, a questioning attitude, and self-confidence, emphasizing the multifaceted nature of creativity.

Sternberg and Lubart (1999) defined creativity as the ability to produce work that is both novel and appropriate, while Plucker and Beghetto (2004) described it as the interplay of ability and process that results in a product both novel and useful within a social context. Silver (1997) outlined two views of creativity: the classical view, linking it to genius and rare talent (Weisberg, 1988), and the contemporary view, which associates creativity with deep knowledge, extended effort, and adaptability to instruction and experimentation (Sternberg, 1988).

Examining these definitions of creativity reveals both similarities and differences. For instance, divergent production is a common theme, with flexibility and novelty being key traits in Guilford's and Runco's definitions and Silver's summary of perspectives. Both Sternberg and Lubart (1999) and Plucker and Beghetto (2004) consider novelty and usefulness within certain constraints in their definitions of creativity. The classical and contemporary views differ in their characteristics of creativity (e.g., insight and limitation), but agree on the importance of PS and PP as core processes in creative ability (Silver, 1997).

Definitions of Mathematical Creativity

Similar to defining creativity, researchers (e.g., Haylock, 1987; Leikin, 2009; Liljedahl & Sriraman, 2006; Mann, 2006; Sriraman, 2005) argue that there is no universal definition of mathematical creativity and that mathematical creativity is multifaceted in nature (Shriki, 2010). Haylock (1987) suggests that "any definition of mathematical creativity in schoolchildren must refer to both mathematics and creativity" (p. 62). Emphasizing mathematical thinking and processes exclusively can lead to questions about the creative aspects of the thought processes involved. Conversely, focusing predominantly on creativity, such as emphasizing novelty, might neglect the importance of validating the products within the mathematical context.

Haylock (1987) noted key features of mathematical creativity: overcoming mental fixations, and divergent production evaluated in terms of fluency, flexibility, originality, elaboration, and sensitivity. These features align with general creativity traits but are specific to mathematical contexts. Ervynck (1991) defined mathematical creativity as the ability to make non-algorithmic, divergent decisions that inherently involve choices, such as defining concepts or formulating and proving theorems. Building on Ervynck's definition, Sriraman et al. (2011) defined mathematical creativity as "the ability to solve problems and/or to develop thinking [cognitive] structures about a mathematical concept or set of concepts considering both the historical development of a concept as well as its logico-deductive framework" (p. 121). Using this description, changing or modifying a network of mathematical concepts is considered creative work. Other definitions of mathematical creativity include that of Liljedahl and Sriraman (2006). Researchers have also considered generalization as a form of creative mathematical activity (Ervynck,1991; Sriraman, 2003), whereby an individual develops conjectures, formulates and tests a hypothesis, then generalizes some mathematical fact or property.

Common themes from the above definitions of mathematical creativity include ability to produce something new, and flexibility in breaking out of the norm (e.g., overcoming fixations, conventions, and non-algorithmic thinking). These definitions also align with the domain specificity of mathematical creativity. However, akin to researchers' approaches to general creativity, there is no agreed upon definition of mathematical creativity despite compatibilities in terms of characteristics of mathematical creativity in the different definitions (Sriraman et al., 2011). As a point we expand upon below, we note that despite the domain specificity of mathematical creativity, the definitions largely focus on generalized characteristics or forms of mathematical activity, as opposed to being tied to differentiated, detailed, and particular meanings for a mathematical concept or idea.

Perspectives on Creativity and Mathematical Creativity

This section shifts from definitions to perspectives on creativity and mathematical creativity. We first explore six approaches to studying creativity, (i.e., mystical, pragmatic, psychodynamic, psychometric, social-personality, and cognitive) (Sternberg & Lubart, 1999). We then examine the process and product views of mathematical creativity and discuss its domain-specific nature.

Approaches used to study creativity. Among the approaches researchers have used to understand creativity includes the mystical, pragmatic, psychodynamic, psychometric, social-personality, and cognitive approach (Sternberg & Lubart, 1999). The mystical approach considers creativity to stem from divine intervention or a spiritual process, where the creative individual is seen as an empty vessel that a divine entity fills with inspiration. The pragmatic approach is "concerned primarily with developing creativity, secondarily with understanding it, but almost not at all with testing the validity of their ideas about it" (p. 5). Sternberg and Lubart (1999) argue that the mystical and pragmatic approaches hinder the scientific study of creativity because individuals who take such an approach have the perspective that creativity cannot be studied scientifically. Guilford (1950), in support of a scientific approach to creativity, argued for the need to evaluate creative work for it to be considered realistic or acceptable.

In the psychodynamic approach to studying creativity, creativity is considered to originate from the tension between conscious reality and unconscious drives. This approach can be considered as the first major 20th century theoretical approach to studying creativity (Sternberg & Lubart, 1999). The psychometric approach pertains to quantifying creativity with the aid of paper and pencil tasks. According to Sternberg and Lubart (1999), an advantage of the psychometric approach to studying creativity is that its tasks are easy to administer and score, and can also engage non-eminent individuals in research. The cognitive approach emphasizes understanding the "mental representations and processes underlying human thought" (Sternberg & Lubart, 1999, p. 7). Lastly, the social-personality approach perceives creativity as resulting from personality and motivational factors, and the sociocultural environment.

Although the cognitive approach described above is on the study of creativity in general, several researchers (Kattou et al., 2013; Leikin et al., 2013; Liljedahl, 2013; Pitta-Pantazi et al., 2013; Singer et al., 2017) in mathematics education have adopted a cognitive approach to study mathematical creativity and/or discuss creativity in mathematics education. Their approach to studying mathematical creativity is similar to that described by Sternberg and Lubart (1999), in which the focus is on mental processes underlying the mathematical creativity of individuals. Given our interest in the ways of reasoning that underpin mathematical creativity, we adopt a cognitive approach to provide a theoretical and methodological contribution to studying mathematical creativity. Accordingly, we next focus on two predominant views within the cognitive approach.

The process view of mathematical creativity. A process view of mathematical creativity focuses on understanding the cognitive processes involved in creative thinking (Haylock, 1987). Creative processes have been modeled in stages, such as Wallas' (1926) model, which includes preparation, incubation, illumination, and verification. Similarly, Cropley and Urban (2000) expanded this model to include the additional stages of information, communication, and validation. In the preparation stage, the problem solver thoroughly explores the problem to understand it. Incubation involves a period of unconscious thought, leading to the illumination stage where a sudden idea emerges. The verification stage involves testing and refining the idea (Wallas, 1926).

Researchers have tested these models in mathematical contexts. Sriraman (2004) found that Wallas' stages were evident in the creative processes of mathematicians. Liljedahl (2013) noted that the emotional aspects of the AHA! experience distinguish illuminations from other mathematical experiences. Schindler and Lilienthal (2019) observed that existing models, including that provided by Wallas, did not fully explain a student's creative process, particularly the absence of incubation. They attributed this observation to the nature of the problem and environmental conditions, suggesting students' creative processes differ from those of experts.

Understanding students' creative processes in mathematics is crucial for several reasons, including developing instructional approaches that foster creativity. Schindler and Lilienthal (2019) emphasized that investigating the creative process helps identify ways to enhance creativity in students, preparing them for future societal demands for creative thinkers and problem solvers. Pitta-Pantazi et al. (2018) also emphasized the importance of understanding these processes to effectively enhance them, noting that they are inherent within individuals and can be observed through the individual's actions and explanations. Our primary focus is describing cognitive processes in terms of generalized properties that can be applied to various mathematical concepts, while also giving attention to students' creative processes as they relate to the different meanings for a particular mathematical concept.

The product view of mathematical creativity. A product view of mathematical creativity aims at the creation of original and appropriate solutions to mathematics problems (Haylock, 1987). Liljedahl, in Sriraman and Liljedahl (2006) noted that almost all definitions of mathematical creativity point to a product, solution or an outcome, stating, "without a product all that remains is a story; a story without an ending" (p. 19). He argued that "the centrality of product has to do with having a tangible and objective artifact by which to judge the art – by which to judge the *process* – by which to *study* the process" (p. 19).

To assess creative products, researchers (e.g., Bahar & Maker, 2011; Kwon et al., 2006; Lev-Zamir & Leikin, 2011, 2013; Walia, 2012) in mathematics education have used some or all of Torrance's (1974) criteria of fluency, flexibility, elaboration, and originality. This aligns with Haylock's (1987) observation that "assessment of creativity by reference to the creative product has concentrated mainly on the use of divergent production tests, such as those developed by Guilford (1959) and Torrance (1966)" (p. 67). Drawing from Hollands (1972) as cited in Haylock (1987), we define these criteria in Table 1.

Table 1. Definitions of Criteria for Creativity

Criterion	Definition
Fluency	The ability to generate multiple ideas within a short time
Flexibility	The use of varied methods or techniques to problem-solving
Elaboration	Refining and extending methods
Originality	The novelty in students' problem-solving approaches

Domain specificity of mathematical creativity. A prevalent theme in research on mathematical creativity is its domain-specific nature. Piirto (1999) distinguished between general creativity, which involves applying PS across fields, and specific creativity, which relates to the logical deductive nature of a particular domain, such as mathematics. Other researchers (e.g., Baer &, Kaufman, 2005; Huang et al., 2017) also argued that creativity may vary across domains in terms of required knowledge, skills, and dispositions.

Baer and Kaufman (2005) emphasized that "creativity can't be entirely free floating and abstract, but must touch down and embrace some content" (p. 158). Chamberlin and Moon (2005) also highlighted that mathematical creativity is distinct from creativity in other disciplines due to its domain specificity. Accordingly, Huang et al. (2017) established that mathematical knowledge positively affected performance in a mathematical creativity test. Other supporters of the domain specificity of mathematical creativity include Feist (2005), and Leikin (2013).

Building on this, we view mathematical creativity as domain-specific to mathematics. Creativity requires knowledge, skills, and expertise within a specific content domain (Baer, 2012), and mathematics is no exception. Taking a cognitive lens of studying creativity, we hypothesize that the schemes and operations in mathematical creativity align with those in mathematics, reinforcing its domain specific nature. Extant research has situated the domain specific aspect of mathematical creativity in terms of generalized processes or forms of reasoning specific to mathematics. These processes are general in the sense that they can be applied across multiple topics. In our work, we narrow this focus further by defining mathematical creativity in terms of the particular schemes and operations that form a meaning for a specific topic. We elaborate on this specification in the next section by further clarifying our cognitive and epistemological perspective.

Perspectives Informing the Beyond Conceptions Approach

In this section, we detail our innovative approach to study PSTs' mathematical creativity beyond their conceptions of mathematical creativity. We first present the epistemological perspective, radical constructivism (von Glasersfeld, 1995), that helps us operationalize mathematical creativity in terms of the schemes and operations (e.g., meanings) for a particular topic (e.g., quadratic growth). We then return to a particular framework for mathematical creativity, adopted from Lithner (2008), and connect that framework to the radical constructivist perspective.

Radical constructivism. As a theory of knowing, radical constructivism provides a guide to the kinds of explanations that researchers produce regarding an individual's meaning in context, such as in a mathematical context. In adopting a radical constructivist perspective, it is important to first note that we intend not to use it to define mathematical creativity in and of itself, but rather to drive and form the kinds of descriptions that we can provide, particularly about the mental operations underpinning an individual's mathematical creativity in the moment. The role of radical constructivism in theories of mathematics education "[is not] to explain phenomena or to prescribe particular actions. Rather, [its] function is to constrain the types of explanations we give and to frame our descriptions of what needs explaining" (Thompson, 2000, p. 417).

Informed by Piaget's genetic epistemology, von Glasersfeld (1995) introduced radical constructivism as a theory of knowing that follows two fundamental principles:

1. Knowledge is not passively received either through the senses or by way of communication; knowledge is actively built up by the cognizing subject.
2. The function of cognition is adaptive, in the biological sense of the term, tending towards fit or viability; cognition serves the subject's organization of the experiential world, not the discovery of an objective ontological reality (von Glasersfeld, 1995, p. 51).

The first principle informs our work in two primary ways. First, we assume that an individual is actively constructing knowledge as they engage in mathematical activities such as PS and PP. That is, PS and PP is more a construction (or reconstruction) of ways of operating than it is an application of ready-made knowledge. Second, because knowledge is activity built, we consider an individual's independent existing knowledge as critical during a mathematical activity because their available ways of operating can constrain or afford their understanding and, hence, how they engage in the mathematical activity of PS and PP. In addition, although their available ways of operating constrain or afford the ways in which they engage in, say, PS and PP, individuals can also generate new knowledge during PS and PP by reorganizing their existing schemes (i.e., cognitive structures or mental framework) or constructing novel schemes to form new knowledge.

The second fundamental principle of radical constructivism pertains the construction of knowledge through mental mechanisms and the role it plays in an individual's experience. Specifically, the construction of knowledge is framed in terms of fit and viability, with knowledge serving a functional, working purpose for the cognizing subject, as opposed to the construction of knowledge being some movement toward or approximation of reality and truth. According to von Glasersfeld, a major implication for this knowledge perspective on research and teaching is that, "you cannot transfer knowledge from a teacher to a student as if it were a commodity that you can just transport from one head to another" (Lombardi, 2010, p. 27). This principle also has major implications for our roles as researchers. In following this perspective, the explanations we develop for an individual's ways of operating and mathematical creativity are merely models that provide one viable explanation for their actions. The characterizations we develop as researchers are not a true picture of the individual's knowledge, nor do our characterizations become more "correct" as they become more viable. The models are viable because they offer sensible explanations for the potential reasoning driving the individual's observable behaviors. These models are always subject to change due to additional experiences that engender the need for such change. This need for change often arises through the process of making conjectures and testing those conjectured models. Echoing Ervynck (1991) and Sriraman (2003), just as conjecturing is a crucial aspect of generalizing in mathematics, it is a crucial aspect of building generalized and viable models of mathematically creative activity.

von Glasersfeld's fundamental principles of radical constructivism provide an orienting lens to studying cognition, but the principles alone do not provide the necessary tools to develop characterizations of mathematical reasoning. Several cognitive constructs have been introduced and used extensively by Piaget, von Glasersfeld, and other mathematics education researchers in order to provide the tools necessary to articulate the mental operations of individuals (e.g., Dawkins et al., 2024; Ellis, 2007; Ellis et al., 2022; Fonger & Dogan, 2019; Moore, 2021; Moore & Thompson, 2015; Raskin, 2002; Simon, 2018; Steffe, 2004; Steffe & Thompson, 2000). Major constructs include assimilation, perturbation, accommodation, and equilibration. Assimilation refers to "treating new material as an instance of something known" (von Glasersfeld, 1995, p. 62). Perception and/or conception are the operative processes under assimilation, whereby the cognizing agent adopts an experience into an already existing conceptual structure. When cognizing agents encounter an activity that cannot be assimilated into an existing conceptual structure, or they experience an unexpected result stemming from an act of assimilation, they experience perturbation. This perturbation may lead them to review and consider characteristics that were not considered during assimilation. These new characteristics can prompt a change in the cognizing agent's recognition pattern, enabling them to realize and integrate the new characteristics in the future.

Such occurrences may fundamentally change the existing conceptual structure or lead to the construction of a new structure, resulting in the development of a new scheme (i.e., meaning) by the individual(s) that has/have undergone the change. Developing new schemes constitutes learning and hence, accommodation. Equilibration happens when a cognizing agent reconciles a perturbation, resulting in a state of understanding. Therefore, learning in a particular direction occurs "when a scheme, instead of producing the expected result, leads to perturbation, and perturbation, in turn, to an accommodation that maintains or reestablishes equilibrium" (von Glasersfeld, 1995, p. 68). We contend that this description of accommodation positions learning itself as a creative act, involving the generation of ideas, connections, and understanding novel to the learner. In order to further draw connections between mathematical creativity and this radical constructivist framing of reasoning, we return to a specific framework for the former.

Framework for mathematical creativity. Lithner's (2008) research framework characterizes imitative and creative reasoning, and discusses the origin and effects of different types of reasoning. Our focus is chiefly on the part of the framework that addresses creative reasoning, in alignment with our work on mathematical creativity. Reasoning, in this framework, refers to "the line of thought adopted to produce assertions and reach conclusions in task solving" (Lithner, 2008, p. 257). Reasoning can be viewed as thinking processes, as the product of thinking processes, or both. In the context of task solving, the thinking processes are exemplified by students' flexibility (i.e., the use of varied techniques and ability to adapt without suffering fixation that inhibits progress), their fluency in using varied approaches and adaptations to the situation, and overcoming fixation. The nature of fixation could be content universe fixation, which occurs when an individual "limits the range of elements seen as appropriate for application to one problem" (Lithner, 2008, p. 267), or algorithmic fixation, which occurs when an individual repeatedly uses an initially effective algorithm that eventually becomes an inappropriate fixation.

To characterize reasoning, Lithner considered reasoning as a *product* that appears in the form of a *sequence of reasoning* and outlined the process of solving a task. The process begins when an individual encounters a *problematic situation* and selects a strategy. This initial choice is guided by *predictive argumentation*, in which the reasoner explains why the chosen strategy is appropriate for solving the task. Next, the reasoner implements the chosen strategy, supported by *verification argumentation*, where the reasoner justifies why the strategy effectively solves the problem. Finally, the process culminates in reaching a conclusion. For one to say that a problem solver has engaged in creative mathematically founded reasoning, he/she must create the reasoning to undergo transition.

For the created reasoning to be considered as *creative mathematically founded reasoning* (CMR) it must meet all of the following criteria according to Lithner:

1. Novelty. A new (to the reasoner) reasoning sequence is created, or a forgotten one is recreated.
2. Plausibility. There are arguments supporting the strategy choice and/or strategy implementation motivating why the conclusions are true or plausible.
3. Mathematical foundation. The arguments are *anchored* in intrinsic mathematical properties of the components involved in the reasoning (Lithner, 2008, p. 266).

> *anchoring*, … refers not to the logical coherence of the warrant but to its fastening in data in relevant mathematical properties of the *components* one is reasoning about: objects, transformations, and concepts. The *object* is the fundamental entity, the 'thing' that one is doing something with, e.g., numbers, variables, functions, diagrams, etc. A *transformation* is what is being done to an object, and the outcome is another object.… A *concept* is a central mathematical idea built on a set of objects, transformations, and their properties, e.g., the function or infinity concept (p. 261).

In connection to the mathematical foundation and anchoring, the relevance of a mathematical property depends on context. For example, to determine which is larger between 9/15 and 2/3, how big the numbers 9, 15, 2, 3 are is a *surface* property and is inadequate to solve the problem, while the quotient captures the *intrinsic* property. Focusing on how big the numbers are would result in selecting 9/15 as the bigger fraction. On the other hand, considering the quotients or comparing the two fractions using other approaches such as converting both fractions into a common denominator would result in a more productive decision. Other important components of Lithner's framework include student competencies (e.g., PS abilities, reasoning abilities and conceptual understanding) and the learning environment in which the competencies are developed are also important.

According to Lithner, a learning environment that promotes creative learning is one in which the teacher provides a suitable problem to the students and refrains from interfering and suggesting knowledge to the student.

Operationalizing mathematical creativity. Our operational definition of mathematical creativity builds on Lithner's (2008) criterion for CMR, emphasizing the ability to engage with mathematical problems in ways that demonstrate flexibility, novelty, plausibility, and a solid mathematical foundation. In general, our perspective on operationalizing mathematical creativity is that everyone (teachers and students) has creative potential that can be nurtured and developed. This has been proposed by other researchers such as Beghetto et al. (2015) and Beghetto and Kaufman (2009). Accordingly, we adopt a relative perspective on creativity (Carreira & Amaral, 2018; Leikin, 2013; Vale et al. 2018), which considers a specific person's discoveries in relation to their previous experiences and peer interactions (Leikin, 2013). By taking this perspective, we avoid limiting an individual's mathematical creativity to a certain threshold determined by well-known global breakthroughs or expert knowledge whose knowledge base is different from that of students. For instance, a new reasoning sequence developed by one person might be novel to them, even if it is not new to their peers or mentors. Adopting a relative perspective of creativity ensures that any new reasoning sequence developed by an individual demonstrates the novelty component of creative mathematical reasoning.

Much like most theoretical perspectives, the relative perspective only provides a guiding orientation. To further operationalize mathematical creativity, we integrate the aforementioned constructs from radical constructivism and Lithner's criterion for mathematical creativity. Specifically, we use the cognitive constructs of assimilation, perturbation, accommodation, and

equilibration to define and clarify mathematical creativity constructs and criterion in mathematical activities such as PS and PP.

To begin, recall that Lithner described that the foundation for an act of mathematical creativity as encountering a problematic situation. A problematic situation can have several inter-related dimensions including that of intellectual, affective, or social. In alignment with radical constructivism, and underscoring our interest in mathematical creativity, we describe a problematic situation using the cognitive construct of perturbation. A problematic situation means that an individual encounters an intellectual challenge that stems from a situation in which they experience a state of uncertainty. This state of uncertainty occurs due to their extant ways of operating not solving the perceived problem. This is not to say the individual's extant ways of operating are not relevant to the perceived problem. In fact, their extant ways of operating define the problematic situation via experiences in assimilating to extant schemes. For instance, in encountering a new task, the individual may enact an extant way of operating only to achieve an unexpected result. Or, upon acclimating to a new task, the individual's extant ways of operating may not be sufficient to determine a possible solution path. In each case, the individual's extant ways of operating lead to an experienced perturbation, thus providing a foundation for building new knowledge to reconcile the perturbation. Yet, the extant ways of operating the individual is able to bring forth are not sufficient in and of themselves to reconcile the perturbation, thus satisfying Lithner's mathematical creativity prerequisite of a problematic situation. Figure 1 summarizes the above discussion of the different ways in which perturbations could be experienced.

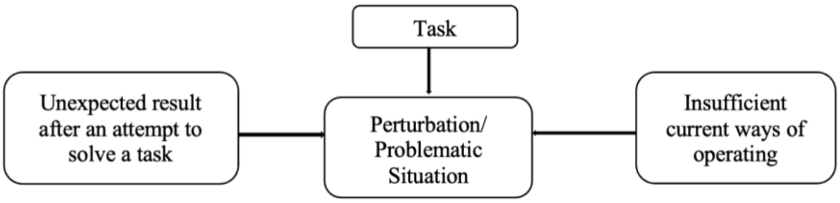


Figure 1. Causes of Perturbation/Problematic Situation

With a problematic situation experienced, the stage is set for Lithner's three CMR criteria. Building new knowledge is connected to the novelty criterion for mathematical creativity, as novelty refers to the creation of a new reasoning sequence or the re-creation of a previously forgotten sequence of reasoning (Lithner, 2008). Drawing on the cognitive constructs of radical constructivism, the mechanism by which new knowledge is constructed is that of *accommodation*. Importantly, an accommodation is such that the experienced perturbation is reconciled via the modification or creation of new cognitive schemes. This process leads to the reestablishment of equilibration through an act of assimilation afforded by the new or modified schemes. This process of building new knowledge through accommodation is learning; the individual's new way of operating, which is neither algorithmic or memorized, is such that (s)he no longer experiences a constraint in the present context. Before transitioning our focus to flexibility, we note that with respect to CMR, an accommodation need not be generalized or permanent. Following Lithner's framing along with the relative perspective, the novelty criteria is with respect to the present problematic situation and is satisfied by an accommodation that reconciles a perturbation and gaining a state of equilibrium in that moment. It could very well be the case that the novel way of operating is not readily available during future problems, as accommodations can be fleeting and take repeated enactments to become stable knowledge resources (Steffe & Olive, 2009).

Turning our attention to flexibility, recall that it involves the ability to embrace diverse approaches and adaptations in a given situation (Lithner, 2008). In order to clarify what classifies as a diverse approach or adaptation, we relate flexibility to engaging in mental actions to establish equilibrium, consider multiple ways of operating, and possibly explore connections between those ways of operating. For instance, the multiple ways of operating could involve the student enacting several distinct meanings to take different approaches to solving a task. A student may or may not relate those different meanings. Or, the student could constitute *representational flexibility*, whereby a student illustrates the growth of quantities such as area and height of growing figures in multiple ways including graphs, tables, and growing shapes without being fixated to one way of representing growth. The student could also enact actions across the various representational systems (e.g., highlighting same amounts of growth in area across the graphs, tables, and growing shapes and make connections between the different representations) such that the student understands the actions as entailing the same mathematical properties.

Shifting our focus to plausibility and mathematical foundation, plausibility refers to the arguments supporting an individual's choice and/or implementation of a strategy (Lithner, 2008), while mathematical foundation focuses on whether the provided arguments are grounded in the intrinsic properties of the components involved in reasoning (Lithner, 2008). As individuals engage in cognitive processes, they enact various mental operations. The foundation of the enacted mental operations may vary. For instance, the arguments could be founded on memorized associations and facts or indexical operations (Piaget, 1970), which are concrete, perceptual, and entail understanding temporal relationships. In the context of quadratic growth, an indexical operation might entail recognizing a pattern in the sequence of numbers and associating it with memorized physical properties of points on a graph. For example, a student might notice that for "quadratic growth", as the student moves to the right on the x-axis of a cartesian plane, the corresponding y-values physically move up more rapidly. On the other hand, the arguments could be rooted in logico-mathematical operations (Piaget, 1970), which are abstract and involve logical structures. In the context of quadratic growth, a logico-mathematical operation might involve understanding and reasoning about the algebraic form of a quadratic function, $f(x)=ax^2+bx+c$, and applying this understanding to determine the nature of a function's growth. For instance, a student might infer that because the coefficient of x^2 (i.e., a) is positive, the rate of change (derivative) of the function increases as x increases, leading to the y-values on the graph increasing by constantly increasing amounts for unit changes in x. Here the focus foregrounds the mathematical properties and their implications, as opposed to physical properties of the Cartesian plane and memorized associations. For reasoning to be plausible and mathematically founded, we relate the way of operating to that which is logico-mathematical in nature, which provides room for abstract thinking and constructing of new mathematical knowledge. In Table 2, we summarize the connections between the constructs of mathematical creativity and the cognitive constructs derived from radical constructivism to aid the operationalizing of mathematical creativity in terms of cognitive processes.

Table 2. Connecting mathematical creativity constructs and radical constructivism constructs

Creativity Construct	Adaptation from Radical Constructivism
Problematic Situation	Encountering a perturbation
Flexibility	Engaging in cognitive processes which involves contemplating multiple ways of operating, representations, and their interconnectedness
Novelty	Constructing new knowledge through accommodation and in response to perturbation
Plausibility and Mathematical Foundation	Characteristics and properties of arguments and of the mental operations and their interconnectedness to form the enacted, and possibly logico-mathematical meaning

Investigating the Operationalization of Mathematical Creativity through PSTs' Mathematical Activities

So far, the narrative we have provided in blending the two theories (i.e., from a meanings and creativity perspective) is theoretical. The question is whether this blending of two theories works; does it provide productive and useful descriptions of PST reasoning and creativity in action? To answer this question, we designed and enacted a methodological approach to investigate secondary PSTs' mathematical creativity. Reflecting our approach to mathematical creativity as domain specific and tied to particular mathematical meanings, our methodological approach first involved targeting a particular topic: quadratic growth. Next, we sought to adopt a methodology that would be compatible with the blended theories. The methodology we chose was a teaching experiment (TE) (Steffe & Thompson, 2000). The main aim of a TE is to experience students' mathematical learning and reasoning firsthand (Steffe & Thompson, 2000), further aligning with our focus on mathematical creativity in action.

From a meaning perspective, a TE served as a suitable methodology because it enabled us to generate and test hypotheses of PSTs' meanings for quadratic growth, and construct PSTs' viable models for quadratic growth. We achieved this objective through conducting series of teaching episodes which highlighted PSTs' ways of operating based on the affordances, constraints, language, and actions of PSTs as they engaged in the PS activities. From a creativity perspective, a TE involved using open-ended problems, which provides a natural setting for PSTs to engage in mathematical creativity. Our use of open-ended tasks was informed by research, particularly the idea that open-ended tasks can elicit, nurture, and strengthen students' mathematical creativity (Haylock, 1997; Kwon et al., 2006; Lee at al., 2003; Levenson et al., 2018; Mann, 2006; Silver, 1997). Returning to our consideration of a methodology that would be compatible with our blended theories, a TE methodology also enabled us to build models of PSTs' mathematical creativity, test them through the cycles of interactions with PSTs throughout the teaching sessions, and relate those models to models of their meanings. Figure 2 summarizes the described design of our study that enabled our application of the blended theories using a TE with open-ended problems.

An additional reason for adopting the TE methodology for exploring our blending of theories to operational mathematical creativity is the flexibility it afforded regarding PS and PP. Traditionally, the TE methodology has been used to build models of students' meanings through a series of teaching sessions. These teaching sessions afford the researcher the opportunity to iteratively build and test these models through strategic task design. In considering the role of the TE methodology for our purposes, we identified the potential of using the iterative design structure to also engage PSTs in PS and PP tasks. Reflecting our underlying epistemology, we conjectured that adopting a TE methodology would enable us to approach PSTs' mathematical creativity during PS and PP tasks as a phenomenon to build models of students' meanings in the hopes of reaching models that we deemed viable. More specifically, we found that the teaching experiment methodology enabled us to iteratively build viable models of PSTs' mathematical ways of operating. These models provided the source material for us to build iterative models of PSTs' mathematical creativity in action.

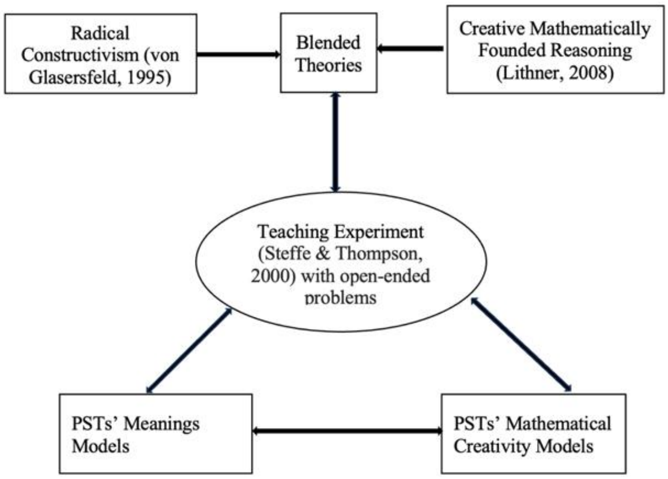


Figure 2. Theoretical and methodological design

In the sections that follow, we used data from our study to illustrate the compatibility between the blending of the two theories and the TE methodology. The TE methodology provided us a productive setting for investigating PSTs' mathematical creativity in action, and we use student actions to highlight how our developed approach helps us understand their reasoning and mathematical creativity in action.

An instance of perturbation, problematic situation, and assimilation. We present the case of Lee and Lilly (pseudonyms), who worked together to explore the growth of a two-dimensional (2-D) growing rectangle adopted from Ellis (2011), with samples in Figure 3. The task was presented on an iPad. In addition to exploring and describing growth, we asked Lee and Lilly to represent growth in different ways. Prior to working on this task, Lee and Lilly had worked on a one-dimensional (1-D) growing rectangle with a constant height and uniform changes in length (Figure 4) and concluded that the growth was linear because the area realized a constant increase for successive, equal changes in length.

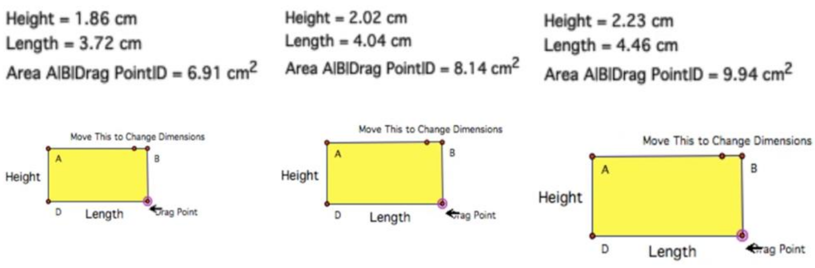


Figure 3. Three sample states of a growing rectangle

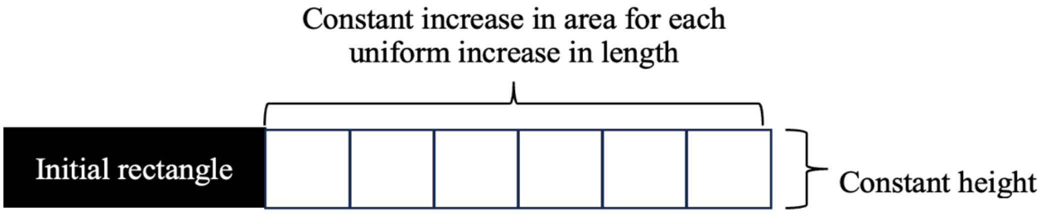


Figure 4. Sample 1-D growing rectangle with constant change in area

After playing the video of the task on an iPad several times to determine its growth, both Lilly and Lee expressed uncertainty about whether the growth in the area of the rectangle followed a "linear" or "exponential" pattern. Excerpts 1 illustrates Lilly and Lee's discussion on the nature of growth.

Excerpts 1. Lilly and Lee's initial encounter with the 2-D growing rectangle

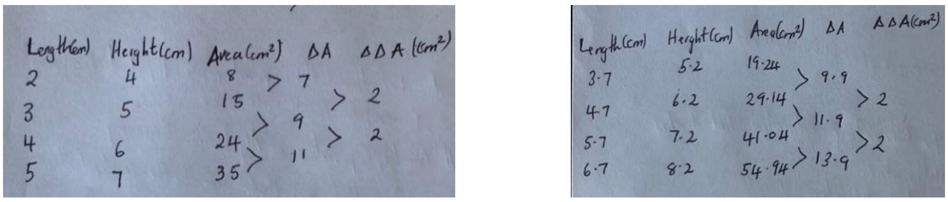
Lilly: Well, based off last time, so last time, it was linear. And that's when the height wasn't changing. But this time the height is changing. So that makes me think it might be either exponential or have a steeper slope. Not sure yet.
Lee: Well, I, if it was… I think, I am thinking it is linear.
Lilly: It is still linear?
Lee: I am thinking linear. If it is exponential what would the rate be?
Lilly: Mmh… [*silence*].
Lee: Well, I am not saying you are wrong. I just don't know.
Lilly: No, I don't know either. I was just thinking of comparing it to before.
TR: Why do you think it will be linear, Lee?
Lee: Well, I don't know because I can't prove it. [*all laugh*]
TR: We are just sketching and then we are going to explore it more.
Lee: Now I think it is exponential. I don't know what I think now.

Considering Lithner's CMR criteria, Lee and Lilly were unsure if the rectangle grew in a "linear" or "exponential" manner, thus indicating a form of uncertainty. Turning our attention to the cognitive constructs of radical constructivism, we infer that their uncertainty indicates a perturbation and a problematic situation that is intellectual in nature. In particular, their discussion suggests their experiencing what Paoletti et al. (2024) defined as a *situational intellectual need*, as they anticipated different potential forms and intensities of growth, specifically linear and non-linear (e.g., "exponential"). According to Paoletti et al. (2024), a student experiences a *situational intellectual need* "when she experiences a (possibly minor) perturbation as she conceives a novel "real-world" situation and subsequently sets the goal-oriented activity of making sense of and mathematizing that situation via a cyclical process of constructing quantities and their relationships" (p. 44). In Lilly's case, her perturbation stemmed from her assimilating the problem based on her previous experience with linear growth, when she compared the 2-D growing rectangle to the 1-D growing rectangle on which she had worked previously. She experienced an unexpected outcome, as she thought that the slope in this case would be steeper. She also expressed uncertainty about the nature of growth due to the situation entailing a changing height. Similarly, Lee oscillated between linear and exponential growth, but remained unsure of the nature of growth. His efforts to assimilate the task into his existing schemes for exponential growth were not fruitful, as he was still unsure of what the "rate" would be. In each case, they considered the relevance of their extant ways of operating for "linear" and "exponential" growth, but neither were satisfactory to the students.

From overgeneralizing to mathematical creativity. In our next example, we examine PSTs' mathematical creativity in conceiving quadratic growth, but at an advanced level and after PSTs had identified that quadratic growth constitutes constantly changing amounts of change. To illustrate, consider a 3 cm by 4 cm 2-D growing rectangle growing uniformly by 1 cm in both dimensions. The areas for the first five stages are 12 cm^2, 20 cm^2, 30 cm^2, 42 cm^2, and 56 cm^2 respectively. The changes in area with respect to changes in length and height are 8 cm^2, 10 cm^2, and 12 cm^2, and 14 cm^2 respectively, indicating a change in area of 2 cm^2. This example illustrates a quadratic growth scenario where the change of area is consistently 2 cm^2 for constant increments in length and height. However, not all quadratic growth scenarios will constitute a constantly changing amount of change of 2 units; it can vary depending on the increments in length and height. We now illustrate how two PSTs encountered and resolved an overgeneralization about quadratic growth, showcasing their mathematical creativity.

Overgeneralizations about constantly changing amounts of change in quadratic growth. As PSTs conceived constantly changing amounts of change in quadratic growth, several overgeneralizations emerged. Here, we illustrate the overgeneralization using another example involving Lee and Lilly. We focus on their overgeneralization that quadratic growth will always have a constantly changing amount of change of 2 units.

Previous to the interaction we illustrate below, Lee and Lilly had worked on a 3 ft. by 4 ft. 2-D growing rectangle task. They increased the length and height of the 2-D growing rectangle by 1 ft. six times, explored each stage of growth, and determined that the difference in the amount of growth in area would be 2 ft^2 for each iteration. They also worked on a task in which they predicted the value of the constantly changing amount of growth in rectangular area when starting with different initial dimensions but maintaining growing both sides by 1 unit each time. Lee determined that this amount for two different initial dimensions of rectangles (a 2 unit by 3 unit and a 3.7 unit by 5.2 unit) was two square units in area per unit change in length and height. Figure 5 illustrates Lee's reproduced work, with the constantly changing amount of change in area indicated in the last column.



(a) (b)

Figure 5. Illustration of Lee's work on 2-D growing rectangles

After determining that the change in the change in area for the two 2-D growing rectangles was 2 cm^2, Lee wondered why the amount was two each time for both rectangles despite their different dimensions. Excerpts 2 shows Lee's expression of his wonder.

Excerpts 2. Lee wondering why the constantly changing amount of change is two.

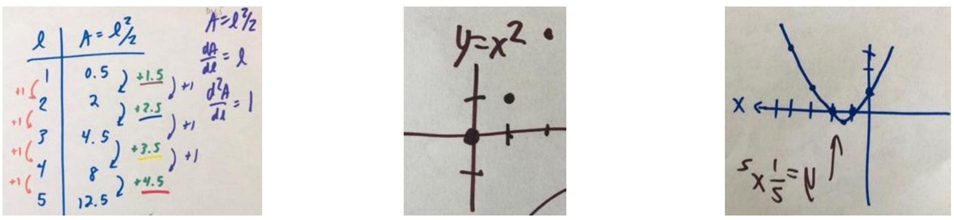
Lee: Why is it plus two? And not like plus three?

TR: That's a good question.

Lee: Why? Cuz I feel like that's how I interpreted the rate of growth, like, not that like, it just it's the same, but I thought it would be different. I thought it wouldn't be plus two, the constant second difference, because it doesn't have to be adding two, it could be minus five, it could be.

Lilly quickly responded, "Because it's a quadratic, and it goes back to calculus. And the derivative of x squared is $2x$. So the slope is two. So it goes up by two every time, I'm serious. Does that make sense?" Lee was not fully convinced. He also added that he did not have a firm understanding of calculus and asked Lilly, "So for a quadratic, you're saying the constant difference will always be plus two?" Lilly affirmed this with a yes. This response showed Lilly's overgeneralization that quadratic growth would always have an amount of change that changed by the same value, what they termed as a constant second difference of two. We hypothesized that Lilly was assimilating the fact that the first and second derivative of x^2 is $2x$ and 2 respectively with respect to x, which she incorrectly generalized to all quadratic growth scenarios. We also conjectured that Lilly's overgeneralization might have stemmed from assimilating the fact that the highest power in all quadratic relationships is two, without attending to how the coefficient of the squared term affects the first and second derivatives or change in the amount of growth.

Despite Lilly's argument, Lee was not satisfied with Lilly's response. He presented a counterexample from a previous exploration where the constant second difference was one square unit (Figure 6). He asked Lilly, "What about this [counterexample], are you saying this [counterexample] isn't quadratic?" Lilly paused for a moment and then admitted, "I think that defeats my point, but it shouldn't." Lilly subsequently inferred that her generalization was incorrect and that a quadratic function would only have a constant second difference of two if the coefficient of the leading term is one, (e.g., $y= x^2$, $y= x^2 + bx + c$, $y= x^2 + c$, where b and c are constants that could take any value). Lilly then engaged in a discussion with Lee (Excerpts 3) to explain her new reasoning.



(a) (b) (c)

Figure 6. Lee and Lilly's illustration of constantly changing amounts of change

Excerpts 3. Lilly realizing her overgeneralization about quadratic growth.

Lilly: Oh, I know why, I know why, I know why. Because this [*the quadratic growth example with a second difference of 1*] one isn't. The parent function wouldn't be y equals x^2. This one's gonna be y equals one half x^2. And so the derivative, that is just x.

Lee: Where did you get the parent function is going to be one half x^2? Where did you get that from?

Lilly: Because normally, because, I need a new piece of paper [*takes a different piece of paper*] let me think of where is original one going? [*gets the paper with the table in Figure 6a that they had been working on*] This one right here. So normally from okay, if you have a parent function if you have y equals x^2 [*draws the cartesian plane in Figure 6b*].

Lee: Yes.

Lilly: The original the, it goes through the origin here. And then you would go one comma one, two comma four [*demonstrates inputs and corresponding outputs for the graph of* $y = x^2$ *and highlights the points as indicated in Figure 6b*]. So from the vertex to the second point, you'd go right one up one.

Lee: Yes.

Lilly: But here, [*demonstrates inputs and corresponding outputs for the graph of* $y = ½ x^2$ *as shown in Figure 6c*] you went, right one up a half, right one up one instead of right one up two.

Lee: Mmh.

Lilly: So we're increasing by half the rate. And so this one right here would be y equals one half x^2, because it's shifted, or what's it called? Compressed?

Lee: Yeah compressed.

Lilly: So the derivative of this, power rule, it would be two times one half here, the slope is one, which is why the second difference is one.

Lee: Mmh. Okay. Okay.

Lilly: So the constant second difference is only going to be two if there's not a…

Lee: [*completes Lilly's statement*] Compression or stretch.

Lilly: Mmh.

Lee: Because if it was a stretch, it will change it too. Wow!

From the above excerpt, we exemplify several meanings and the mathematical creativity demonstrated by Lee and Lilly. Even with his admitted lack of a firm understanding of calculus, Lee challenged Lilly's claim that all quadratic growth scenarios would constitute constantly changing amounts of change of 2. Lee expected that the constantly changing amounts of change could vary and drew on a different scenario of quadratic growth with constantly changing amounts of change of one square unit and not two square units. We note that Lee and Lilly's work illustrate their experiencing a problematic situation. Namely, they initially experienced a situational intellectual need. Here, Lilly experienced a subsequent problematic situation (i.e., a perturbation) when she determined her way of operating to not apply appropriately to a previous situation. Lilly then experienced an "Aha!" moment (Liljedahl, 2013), by realizing why her initial statement was inaccurate for all quadratic growth contexts, and attained a state of equilibrium. This new state of equilibrium was achieved by Lilly reorganizing her mental operations to accommodate a different and expanded way of thinking about quadratic growth that would resolve the overgeneralization that she had made.

When Lilly encountered a perturbation, she considered a different approach to thinking about quadratic functions. Rather than becoming fixated on algorithms or the specific content domain of derivatives, she successfully overcame these potential fixations. Lilly integrated a graphical approach, assimilating the concept of the "parent function" by coordinating values (e.g., "you went, right one up a half, right one up one instead of right one up two") and by imagining graphical changes in a figurative sense (e.g., compression and stretch). This mental shift allowed her to modify her overgeneralization of the function, accommodating new insights and achieving a state of equilibrium in her understanding. This ability to overcome mental fixations and adapt her thinking is a clear indication of mathematical creativity, particularly exemplifying fluency and flexibility as described by Lithner (2008).

Lilly's flexibility was demonstrated not only in her ability to work across various content areas, such as derivatives and transformations (e.g., compressions and stretches) and their algebraic representation, but also in her *representational flexibility* (Carreira & Amaral, 2018). She effectively reasoned about quadratic growth using multiple forms of representation, including tables, graphs, and derivatives, and connected these different representations. Importantly, her representational fluency was founded on her drawing a particular meaning for quadratic functions that foregrounds the growth properties.

As they developed their understanding of quadratic growth, both Lee and Lilly generated mathematical arguments to support their thinking, whether they were in agreement or challenging each other's perspectives. This interaction served as a means for them to ground their reasoning in the mathematical properties of the tasks, such as comparing varying amounts of change. They drew upon their knowledge of calculus, algebra, and graphical knowledge to construct their arguments, thereby demonstrating reasoning that was mathematically plausible and founded. This collaborative process involved PSTs generating new knowledge and revising their existing schemes related to quadratic growth. They accommodated new insights, moving beyond initial overgeneralizations, and ultimately attained equilibrium in their understanding. The result was the learning and development of a new sequence of reasoning about quadratic growth, which showcased novelty and exemplified mathematical creativity.

III. DISCUSSION AND CONCLUSION

We introduced a unique approach to studying mathematical creativity by integrating two theoretical approaches for framing mathematical creativity in action. Together, our framing extends beyond conceptions of mathematical creativity and characteristics of mathematical creativity. Integrating the two approaches was possible because of the compatibility between the approaches. The two approaches are compatible in the sense that they both focus on reasoning in general. The mathematical creativity framework by Lithner characterizes CMR. The constructs from radical constructivism as a theory of knowing highlight mental processes entailed in thinking and learning including perturbation, assimilation, accommodation, and attaining equilibrium. By combining the two, we operationalized a lens for mathematical creativity that focuses on specifying particular mental operations and the creative aspects of those operations.

As indicated in the introduction, most research on mathematical creativity has focused on assessing products of students' work rather than the processes that govern those products. While creative products are important, focusing on mathematical creativity in action, offers a richer understanding of how creative processes unfold in real time. This perspective has practical applications in the teaching and learning of mathematics, such as identifying key factors that influence students' creative thinking (e.g., cognitive and environmental factors), and tailoring learning needs to specific students, by adopting a relative approach to creativity through recognizing that students have unique creative abilities.

Additionally, most frameworks informing research on students' mathematical creativity focus on the characteristics of mathematical creativity such as fluency, flexibility, and novelty. However, these frameworks often do not describe or integrate the mental processes behind these characteristics. By operationalizing mathematical creativity in terms of mental constructs and processes, our approach provides deeper insights into student thinking and supports teachers and researchers in making informed decisions to nurture students' mathematical creativity.

Operationalizing mathematical creativity in terms of mental constructs or processes, has several affordances and implications for teaching, learning, and research. First, we note that by giving attention to building models of the PSTs' thinking, we were able to specify the perturbations, accommodations, meanings, and mathematical properties that underpinned their creative aspects. For example, we were able to specify the ways in which their reasoning was novel or flexible by pinpointing knowledge development and the representations in which it occurred. Or, with respect to mathematical plausibility and foundation, we were able to use our developed models of their thinking as the source material for those characterizations. More broadly, this speaks to the benefit of adopting the TE methodology for studying mathematical creativity in action. The TE methodology enables a researcher to undertake a systematic approach to developing models of student thinking, which can provide the researcher additional data by which to develop models of their mathematical creativity as exhibited during PS and PP.

Secondly, in the context of teaching, if students experience a perturbation during PS, teachers can be intentional about supporting the students rather than falling into the Topaze effect (Brousseau,1997) where teachers think on behalf of the students. Understanding that perturbations or problematic situations precede creativity and presents an opportunity to be creative can allow teachers or researchers to use overcome the experienced perturbations or problematic situations and in so doing encourage them to tap into their creative potential.

Thirdly, taking such an approach can inform curriculum design to enhance students' mathematical creativity. If we aim to understand and support students' mathematical creativity, one way is to integrate characteristics of mathematical creativity with their associated mental processes. In doing so, we take a step further to ensure that students learn how to think creatively about mathematical concepts without shying away from problematic situations when they experience perturbations. Intellectual challenges experienced in learning mathematics would be viewed as drivers for creativity. Relatedly, this can also enhance teachers' metacognition by promoting awareness of their own creativity through professional development that engages them in PS and PP. This metacognitive awareness can help teachers to better support their students' mathematical creativity. Lastly, by proposing this approach to study mathematical creativity, we invite more nuanced studies to explore the complex nature of mathematical creativity, and advance both research, theory and practice in mathematics education.

Limitations of the Approach

While beneficial, our approach has limitations. One limitation in applying the approach to understand individual's mathematical creativity is that in a group setting, attributing mathematical creativity to an individual can be challenging. This is because one student's reasoning sequence can be influenced by the ideas of another student in the group who might be at a different state of knowledge both positively and negatively. For instance, Lilly was at a different level of understanding calculus in relation to Lee. However, because the set-up encouraged exploration and pursuing causes of perturbations, Lee did not agree to everything that Lilly said despite his knowledge base, but rather critiqued Lilly's reasoning. Another key limitation of group settings is that collaboration can obscure individual reasoning and contributions, with dominant voices potentially overshadowing others. This makes it challenging to isolate participants' thought processes, and can limit both flexibility and novelty of students. To address this, teachers and researchers should use follow-up tasks or questions to assess individual mathematical creativity, supplemented by interviews and/or teaching experiments.

Another limitation is that the framework is more applicable during active engagement with mathematical tasks which can be a challenge to its use in the context of for example, assessment of products of student work. Mental processes may not be apparent when one is looking solely on the solutions of a task. One could assess components like flexibility and novelty but constructs such as perturbation, equilibrium, and accommodation may not be apparent. Without an idea of whether a student reached a state of accommodation or not, it can be challenging to determine whether learning took place.

Our findings have limited generalizability due to the focus on TE methodology, which differs from the complexity of larger classrooms. Future research should explore PSTs' mathematical creativity in traditional settings for broader insights.

In conclusion, our approach offers a comprehensive method to study and support mathematical creativity, providing valuable insights for teaching and learning while acknowledging its limitations. By addressing these challenges, we can further enhance the understanding and fostering of mathematical creativity in educational settings.

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